1 The Structure of the Web (35%)

As new links are created and old ones are removed among an existing set of Web pages, the pages move between different parts of the bow-tie structure of the Web.

1. Describe an example of a graph where removing a single edge can reduce the size of the largest strongly connected component by at least 1000 nodes. (Clearly you shouldn't attempt to draw the full graph; rather, you can describe it in words, and also draw a schematic picture.)

2. Describe an example of a graph where adding a single edge can reduce the size of the set OUT by at least 1000 nodes. (Again, you should describe the graph rather than actually drawing it.)

2 Power Laws (30%)

Suppose that some researchers studying educational institutions decide to collect data to address the following two questions:

1. As a function of k, what fraction of UML classes have k students enrolled?

2. As a function of k, what fraction of 3rd-grade elementary school classrooms in Massachusetts have k pupils?

Which one of these would you expect to more closely follow a power-law distribution as a function of k? Provide a brief explanation for your answer.

3 Power Laws & Rich-Get-Richer Phenomenon (35%)

Erdős and Rényi (1960) studied a model of growth for graphs in which, at each step, two nodes are chosen uniformly at random and a link is inserted between them. Do you think power laws and the rich-get-richer phenomena are likely to be observed in these random graphs. Provide a brief explanation for your answer.