Network Basics 2

Advanced Social Computing

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Lecture Topics

- Connected Components
- Breadth-First Search
- Depth-First Search
- Shortest Path Algorithm
  - Dijkstra's algorithm

Connected Components

- Connected component of a graph is a subset of nodes such that:
  - every node in the subset has a path to every other; and
  - the subset is not part of a bigger component.

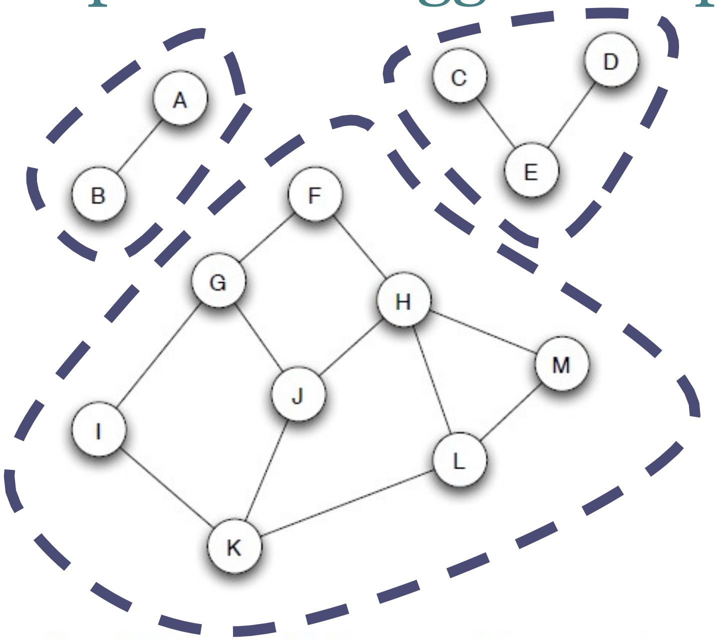

Figure 2.5: A graph with three connected components.

Connected Components

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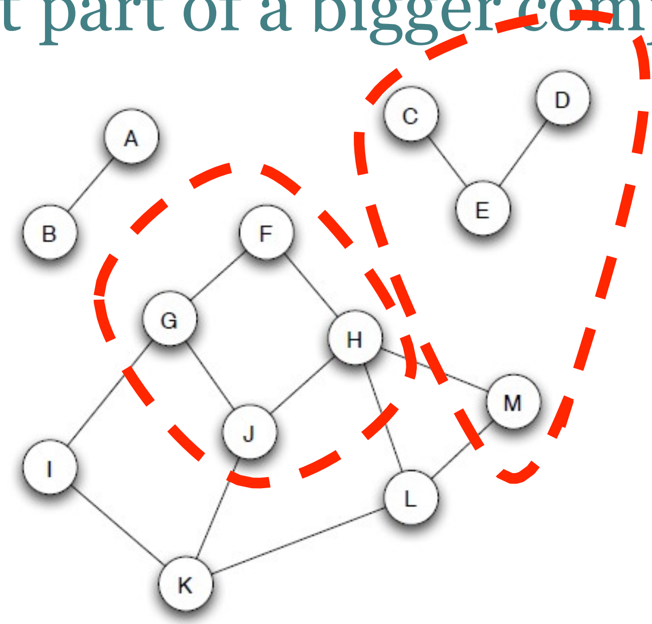
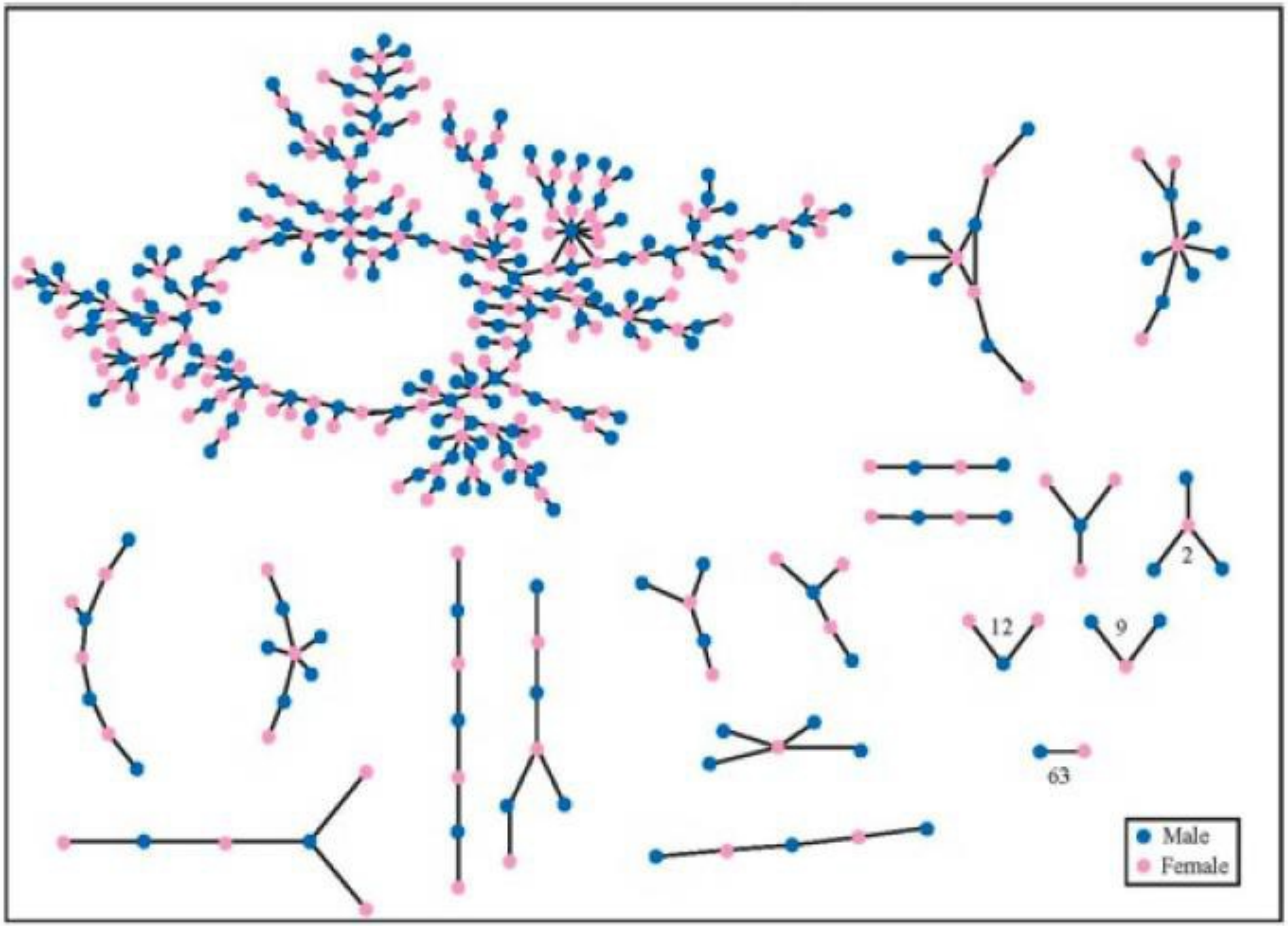


Figure 2.5: A graph with three connected components.

Connected Components- Cnt.



Connected Components- Cnt.

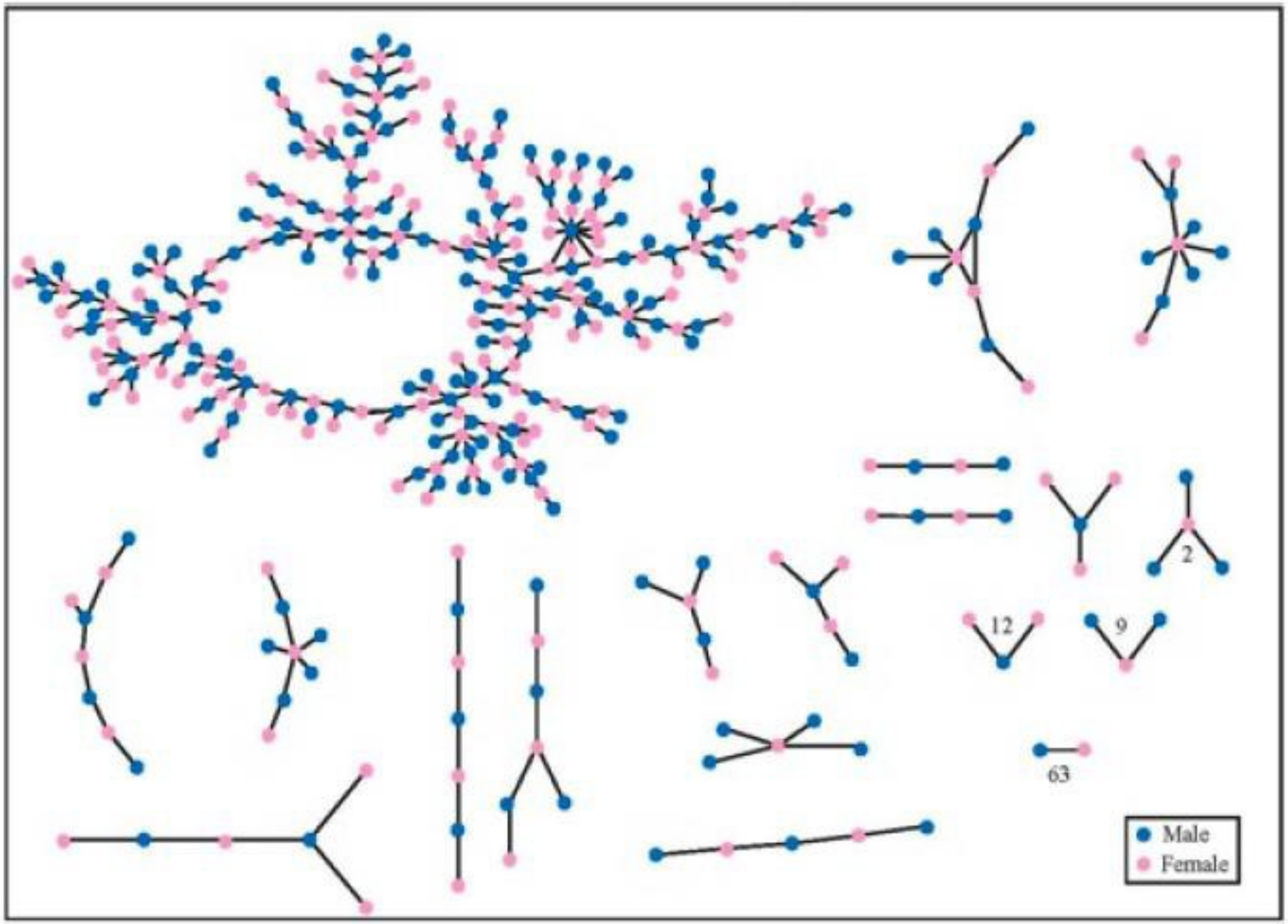


Figure 2.7: A network in which the nodes are students in a large American high school, and an edge joins two who had a romantic relationship at some point during the 18-month period in which the study was conducted [49].

Breadth- Depth-First Search

- General techniques for traversing graphs!
  - Start from a given node *s* (i.e. start node) and visit all nodes and edges in the graph.
- Compute the connected components of graph!
  - Use components to determine whether graph is connected!
    - How?
  - Use components to determine if there is a path btw node pairs!
    - How?

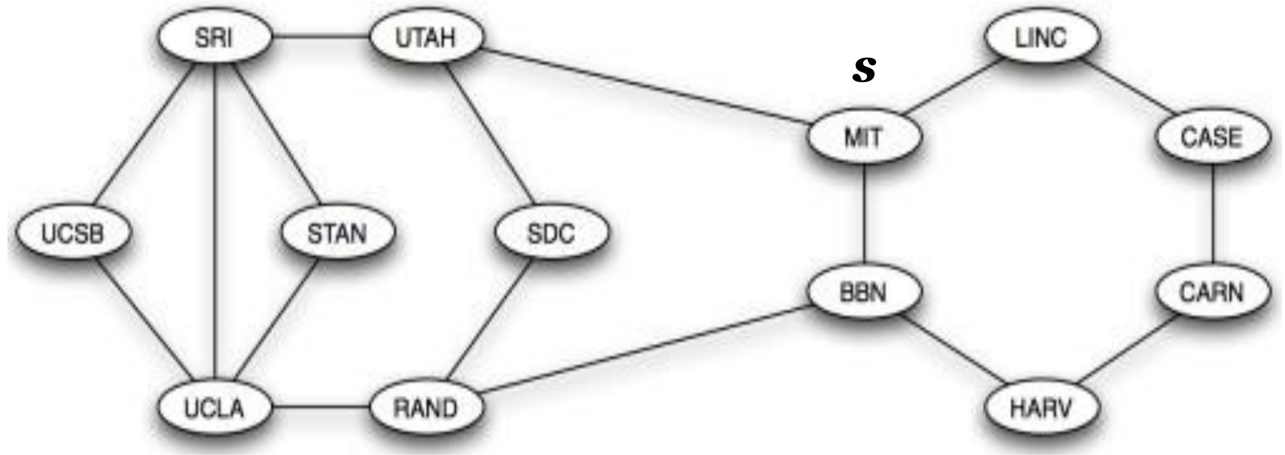
Breadth-First Search

- Start with *s*
- Visit all neighbors of *s*
  - these are called **level-1 nodes**
- Visit all neighbors of level-1 nodes
  - these are called **level-2 nodes**
- Repeat until all nodes are visited.
  - Each Node is only visited once.

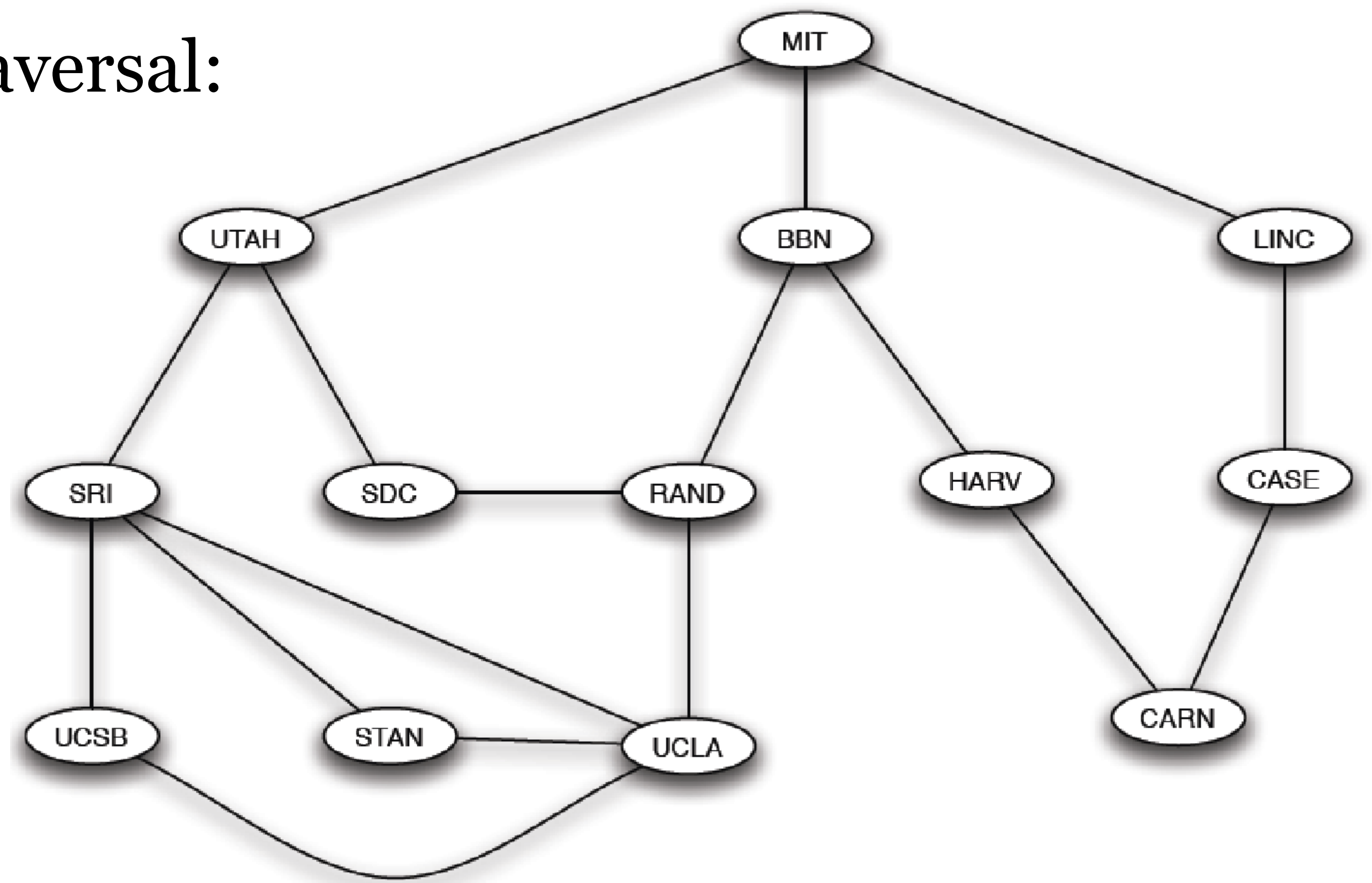
- Key Point:
  - All level-k nodes should be visited before any level-(k+1) node!

Example 1.

- Graph G:

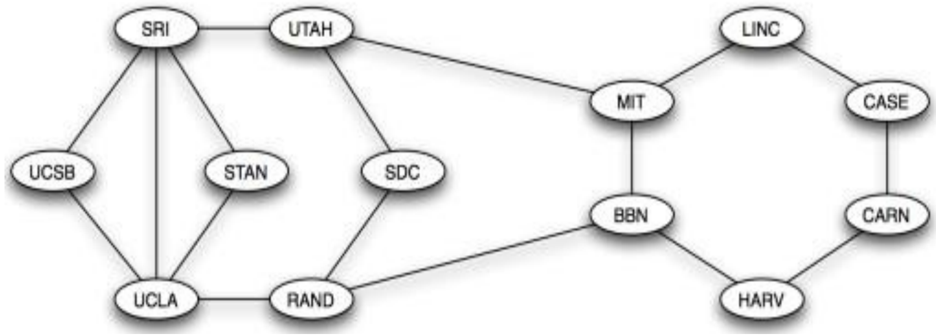


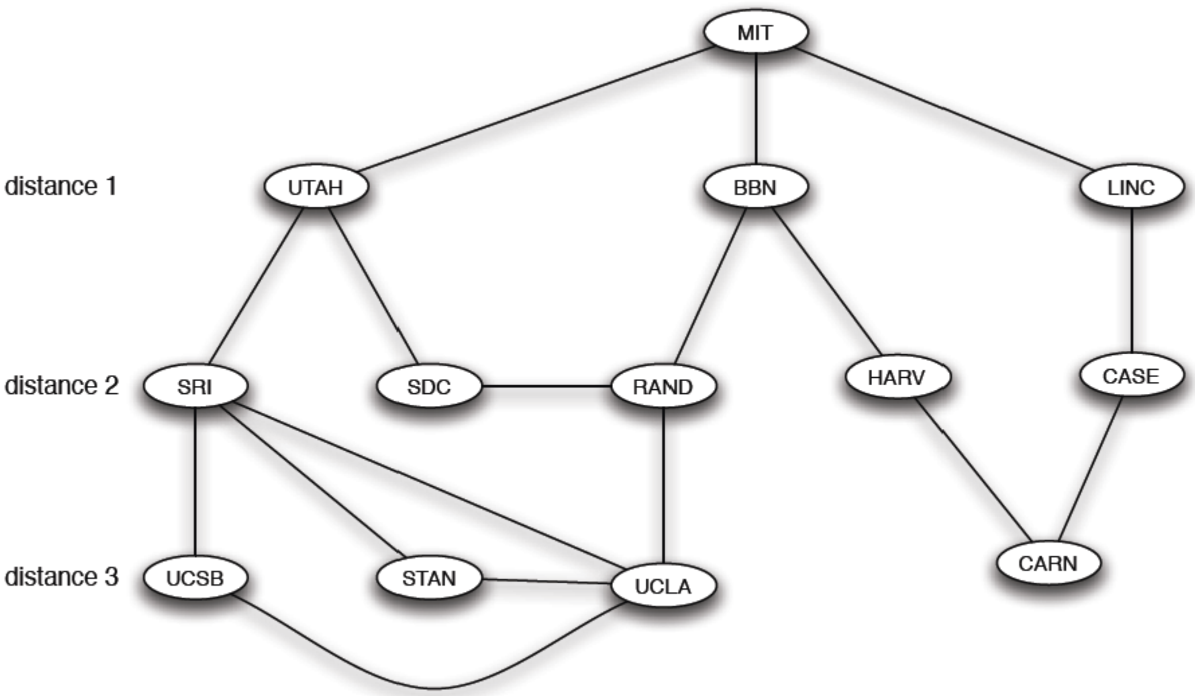
- Its BFS traversal:



Example 1. BFS -Cnt.

- BFS traversal:
  - Distance to root at level-*i*?
  - Components?
    - Connectivity?
    - Paths?





Depth-First Search

- Starts from s
- Explores as far as possible along each branch before backtracking.
  - Visit a neighbor of *s* [say v_1]
  - Visit a neighbor of v_1 [say v_2]
  - Repeat until all nodes are visited.

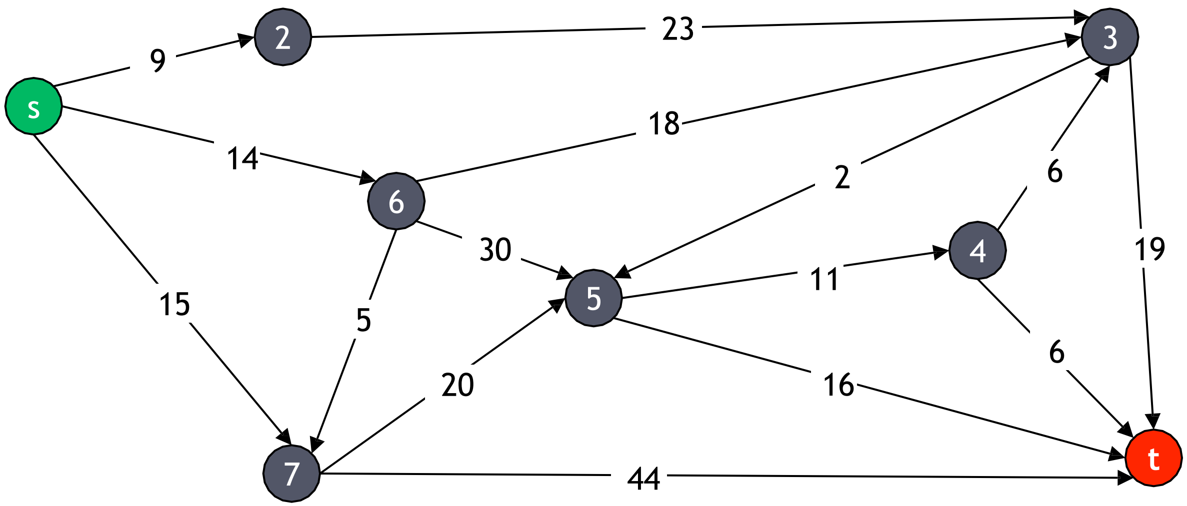
Shortest Path Algorithms

- Given a weighted directed graph and two nodes s and t, find the shortest path from s to t.
  - Cost of path = sum of edge weights in path

Shortest Path Algorithms- Cnt.

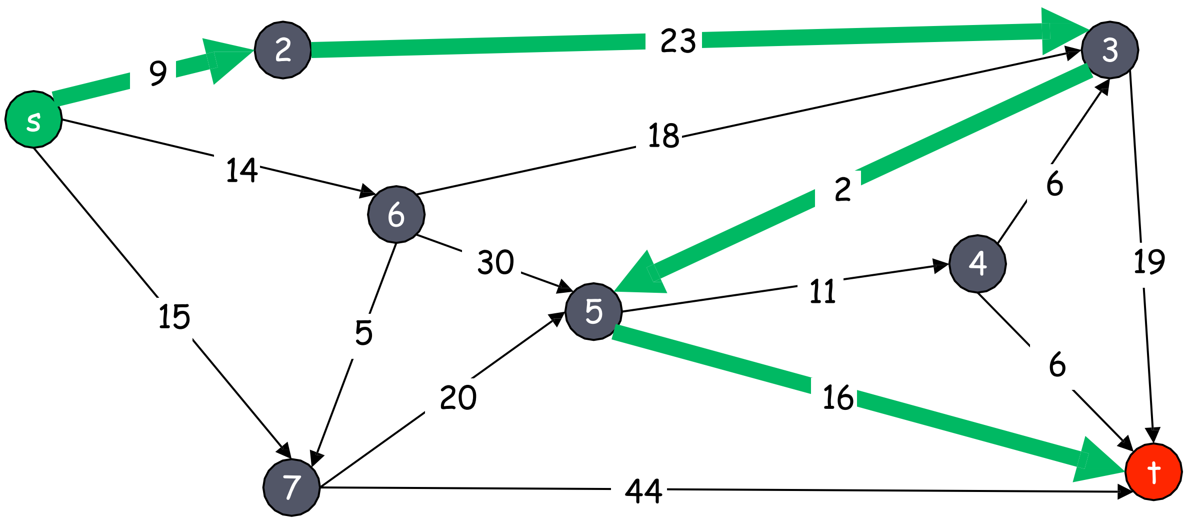
- Dijkstra’s algorithm
- The Bellman-Ford algorithm
- The Floyd-Warshall algorithm
- Johnson's algorithm
- Etc.

Shortest Path Algorithms- Cnt.



- Shortest path from *s* to *t*?

Shortest Path Algorithms- Cnt.



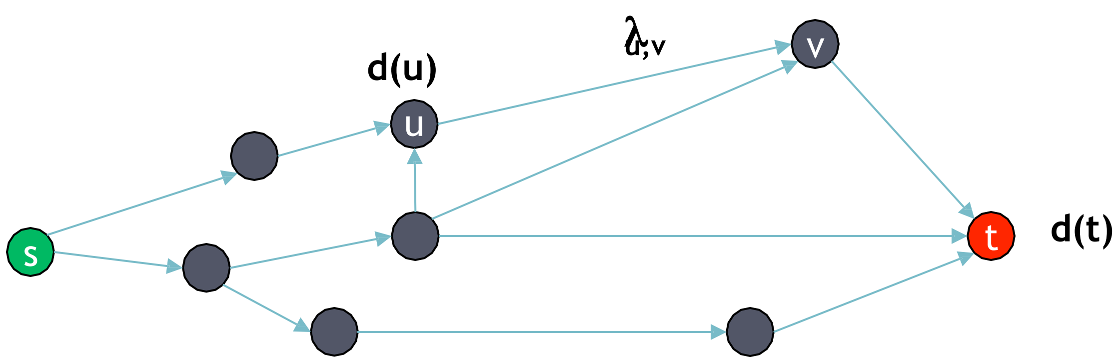
- Shortest Path= s-2-3-5-t
- Cost of path = 9 + 23 + 2 + 16 = 48.

Shortest Path Algorithms- Cnt.

- Applications
  - Small World Phenomenon
  - Internet packet routing
  - Flight reservations
  - Driving directions
  - …

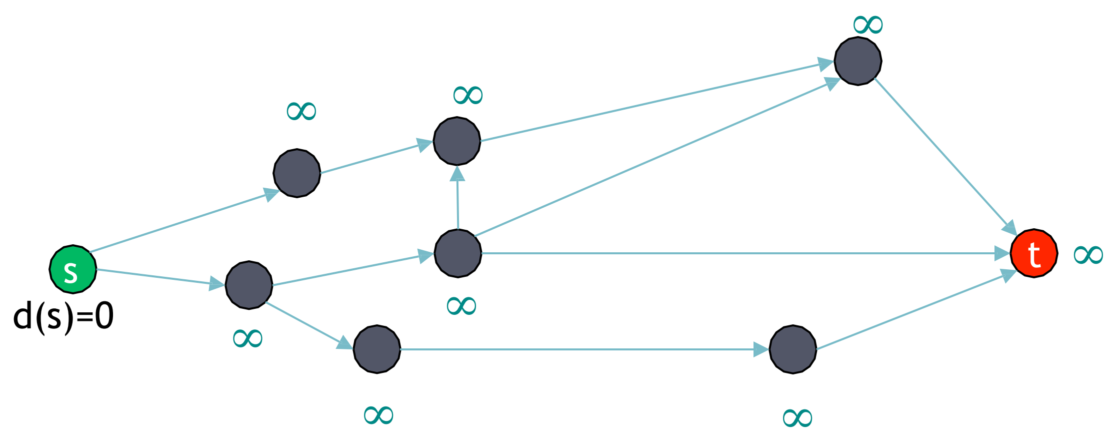
Dijkstra algorithm

- Weighted Directed graph G = (**N**, **E**),
  - *s*: source node
  - *t*: target node
  - $l_{(u,v)}$: weight of the edge btw nodes *u* and *v*
  - *d*(u): shortest path distance from s to u.
    - sum of edge weights in path
- We aim to compute d(*t*)!



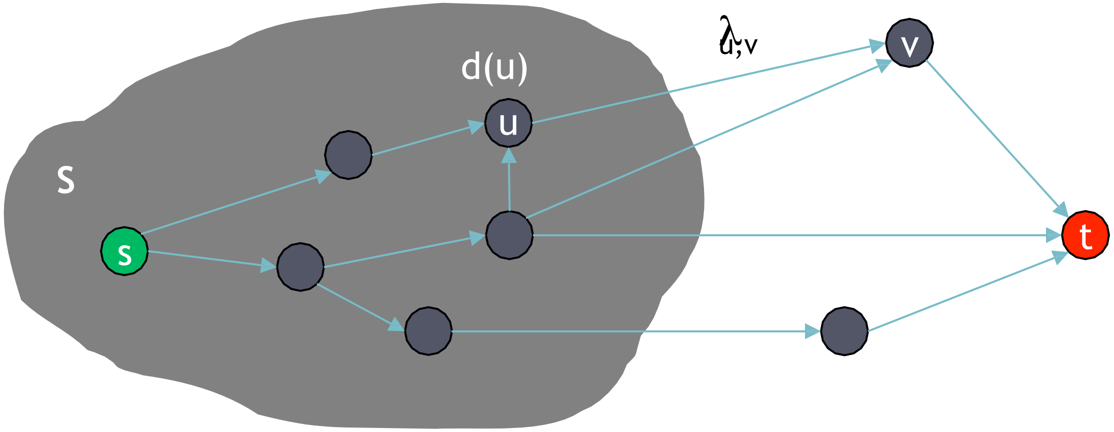
Dijkstra algorithm- Cnt.

- Initialization?
  - $d(s) = 0$
  - $d(u) = \infty$ for all other nodes



Dijkstra algorithm- Cnt.

- To find the shortest path from *s* to *t*:
  - Maintain a set of ***explored nodes*** **S** for which we have determined the shortest path distance from s to any u ϵ **S**.
  - Repeatedly expand **S**.

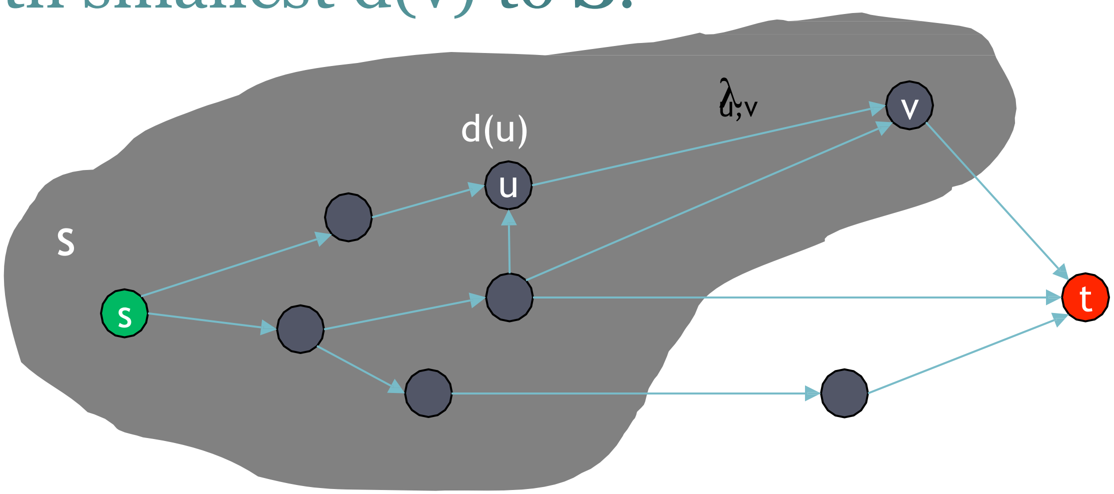


Dijkstra algorithm- Cnt.

- Repeatedly expand **S**?
  - Repeatedly update *d(.)* for the unexplored nodes:

$$\textbf{if } d(v) > d(u) + l_{(u,v)} \quad \textbf{then } d(v) \leftarrow d(u) + l_{(u,v)}$$

  - add v with smallest d(v) to **S**.



Dijkstra algorithm- Cnt.

- $d(s) \leftarrow 0$
- **for** each $v \in \boldsymbol{N} - \{s\}$
  - **do** $d(v) \leftarrow \infty$
- $\boldsymbol{S} \leftarrow \emptyset$
- $\boldsymbol{Q} \leftarrow \boldsymbol{N}$ ▷ Q is a set maintaining $\boldsymbol{N} - \boldsymbol{S}$
- **while** $\boldsymbol{Q} \neq \emptyset$
  - **do** $u \leftarrow$ EXTRACT-MIN($\boldsymbol{Q}$)
    - $\boldsymbol{S} \leftarrow \boldsymbol{S} \cup \{u\}$
    - **for** each $v \in Adj(u)$
      - **do if** $d(v) > d(u) + l_{(u,v)}$
        - **then** $d(v) \leftarrow d(u) + l_{(u,v)}$

Set of explored nodes

Set of unexplored nodes

Returns node u ∈ Q that has minimum d(u)

Add it to explored nodes

Update d(.) for all neighbors of u: this is called **relaxation**!

Example 1.

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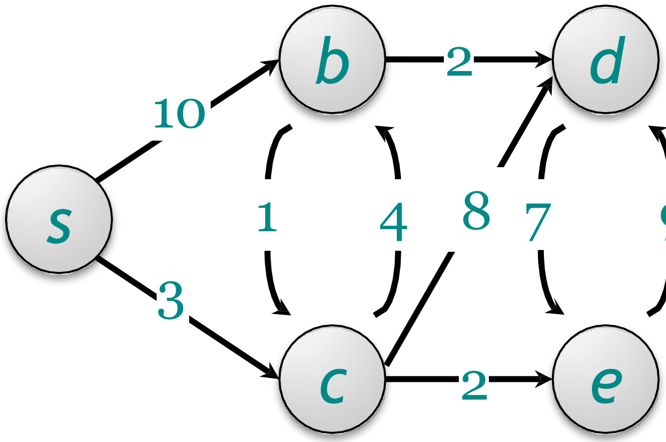

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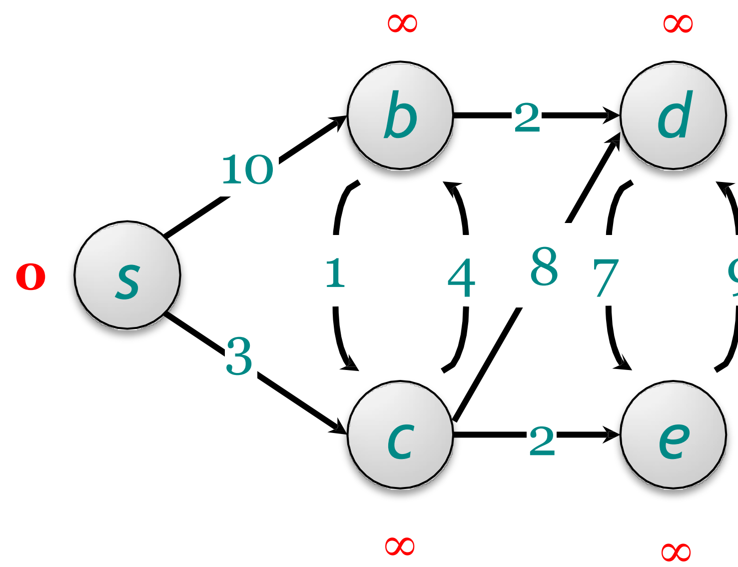

S={}

Q={s, b, c, d, e}

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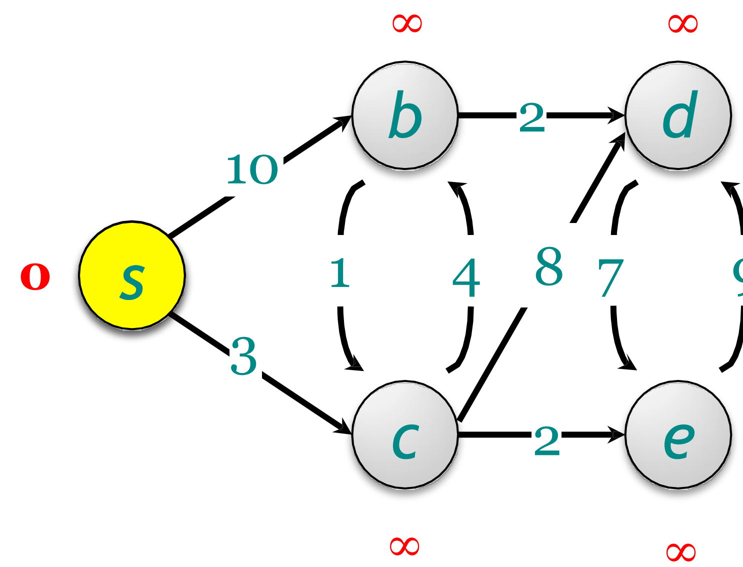

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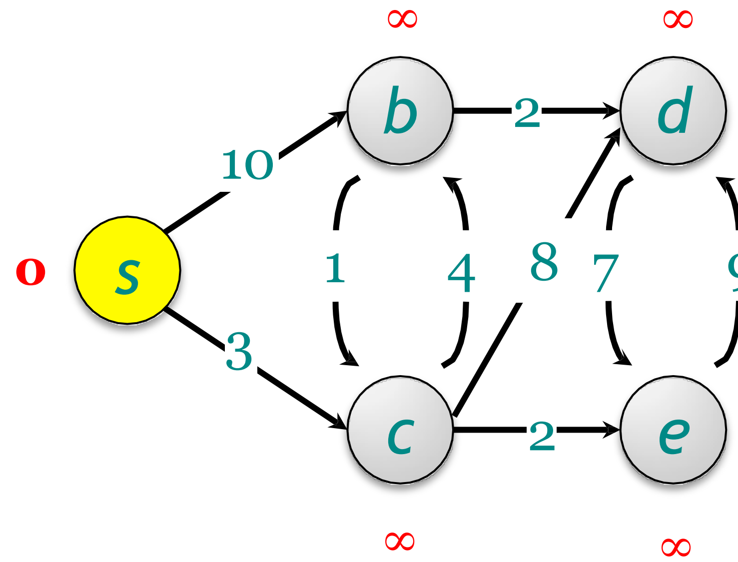

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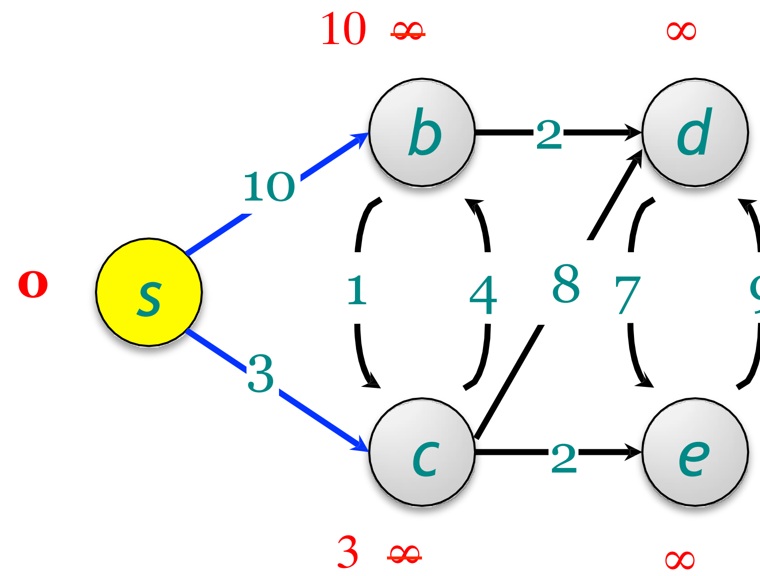

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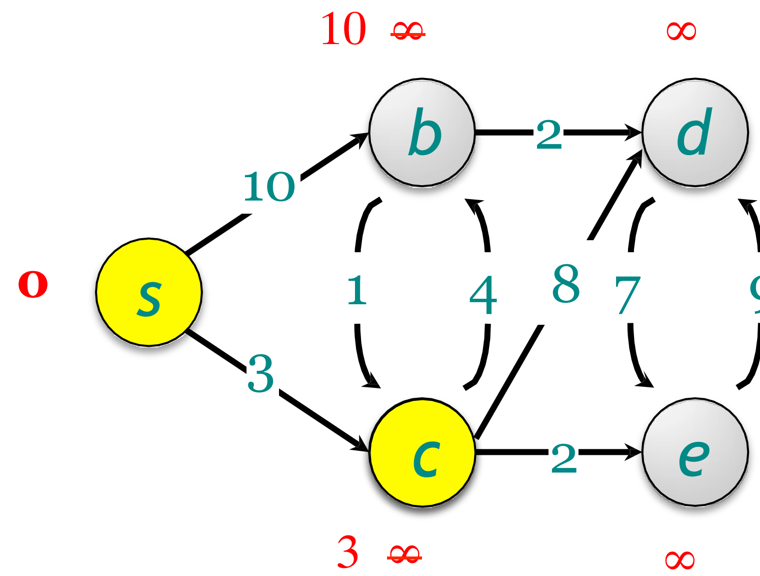

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c

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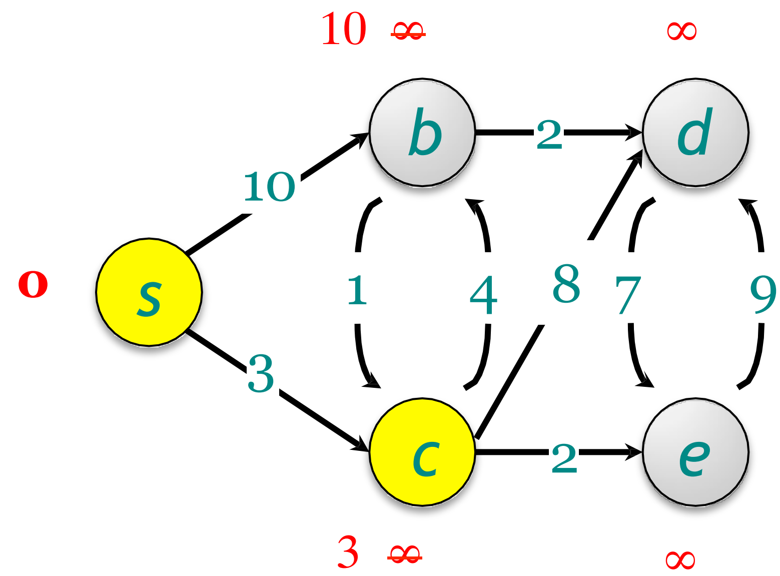

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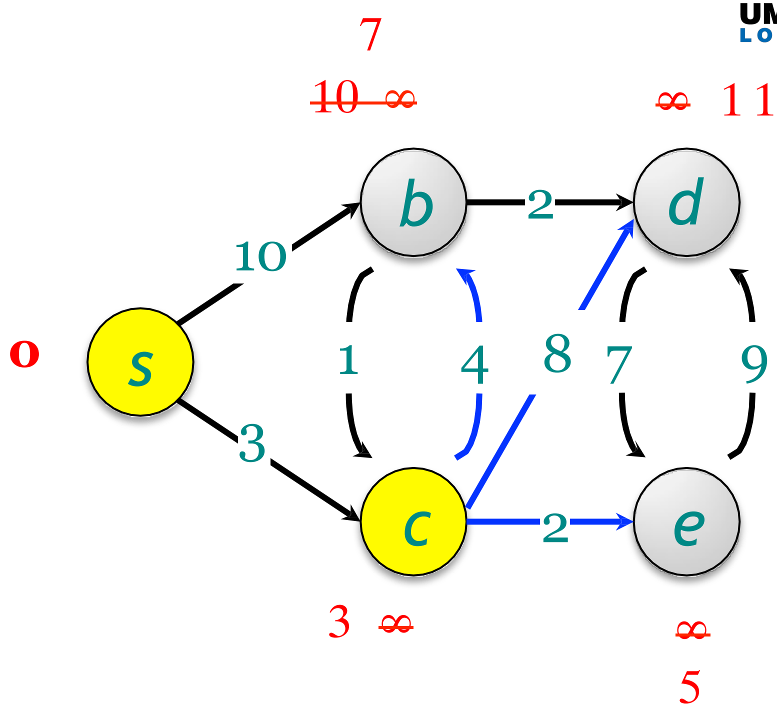

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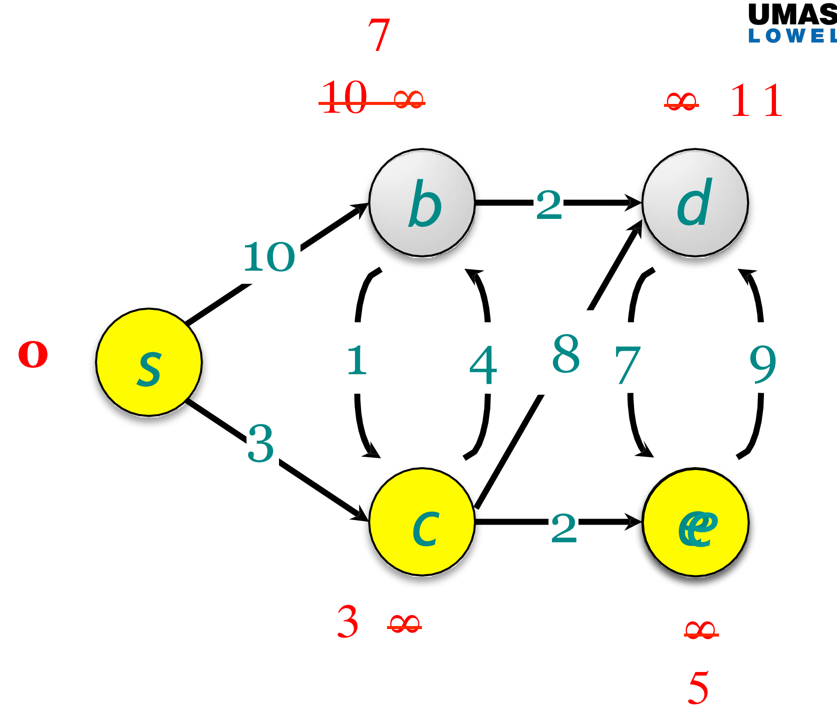

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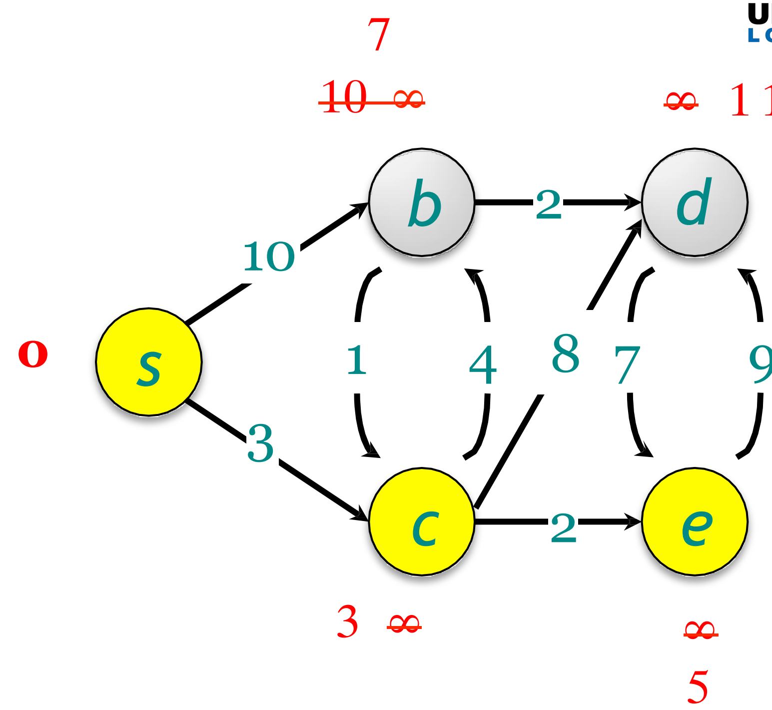

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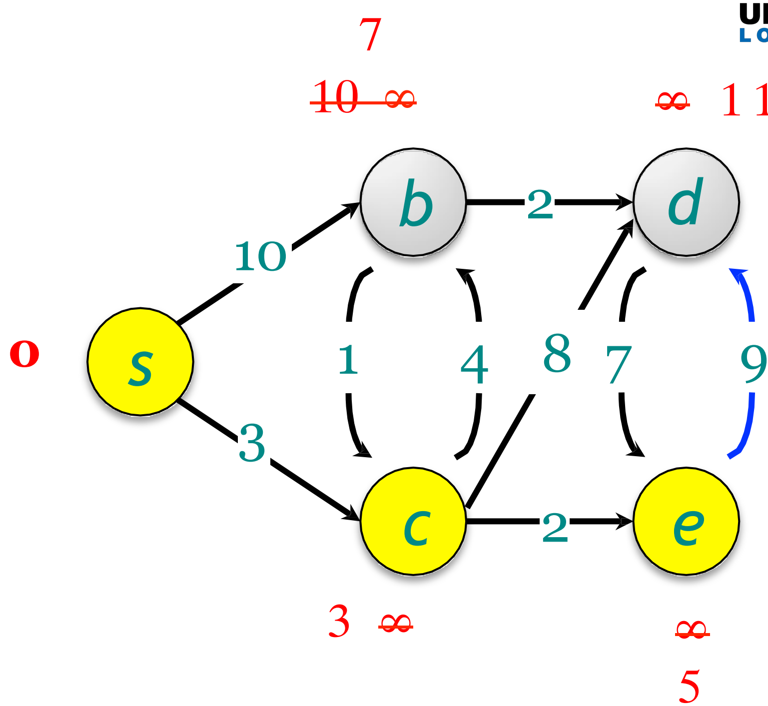

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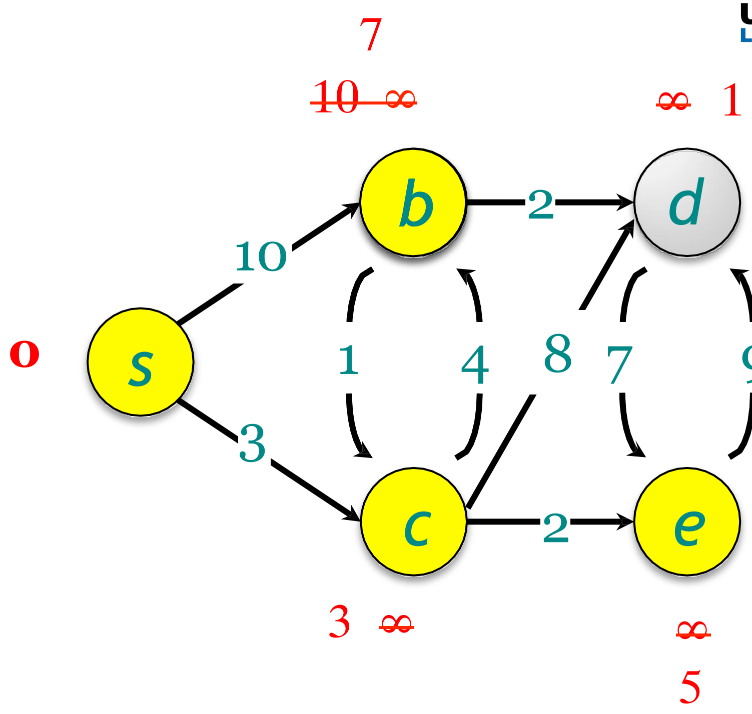

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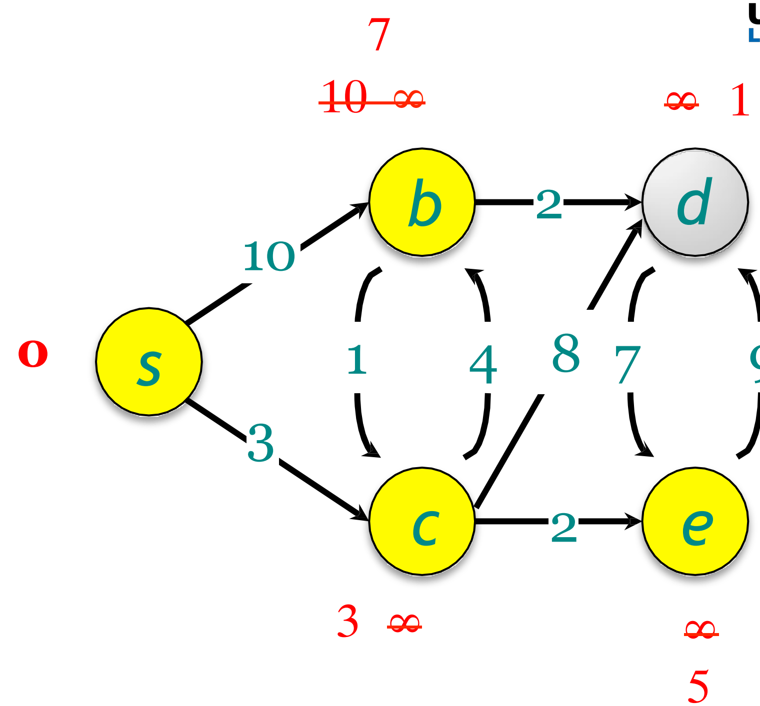

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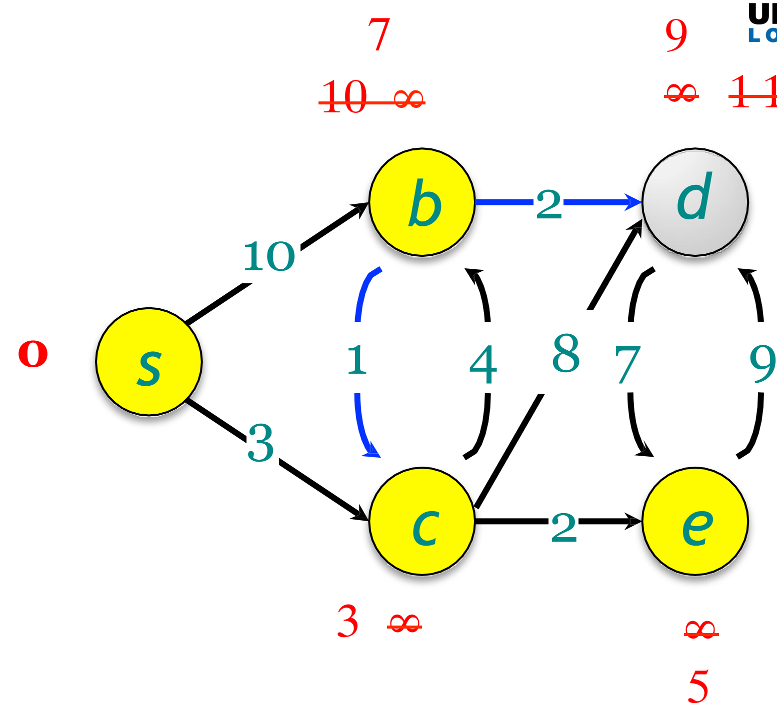

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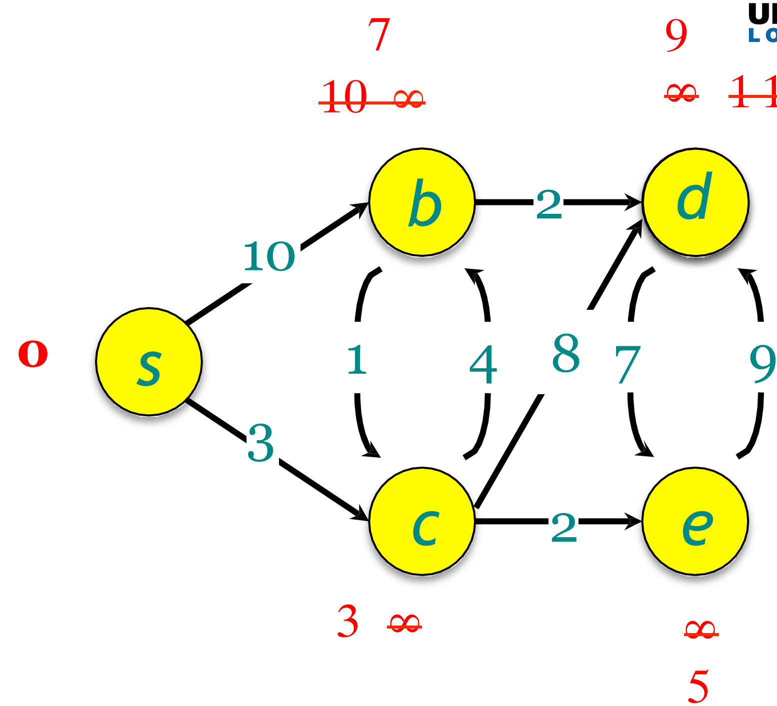

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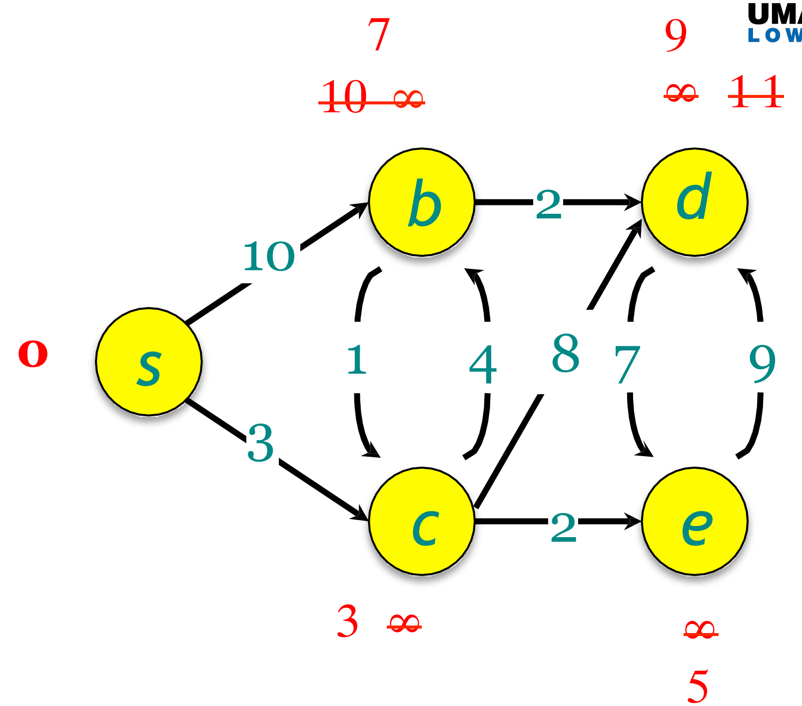

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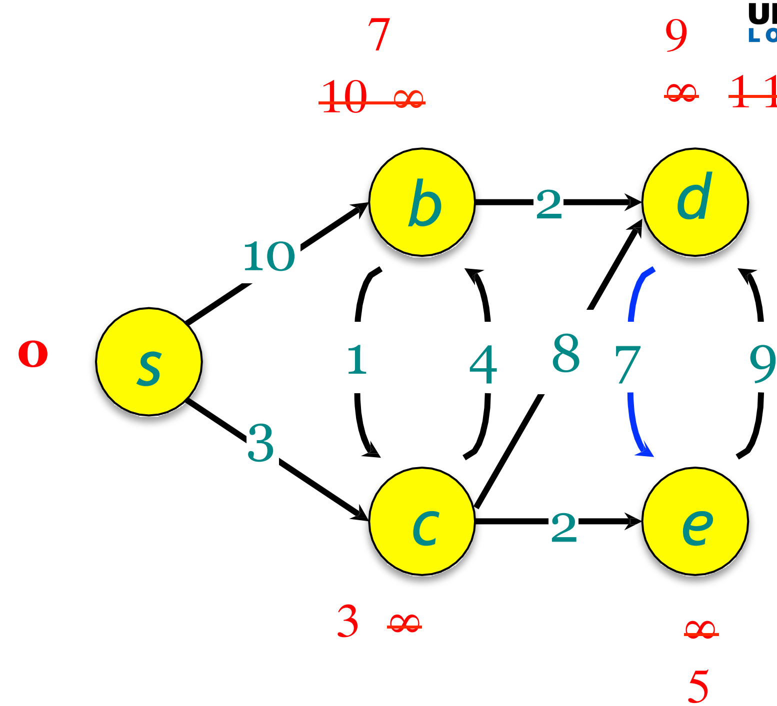
  - **do** $u \leftarrow$ EXTRACT-MIN($\boldsymbol{Q}$)
    - $\boldsymbol{S} \leftarrow \boldsymbol{S} \cup \{u\}$
    - **for** each $v \in Adj(u)$
      - **do if** $d(v) > d(u) + l_{(u,v)}$
        - **then** $d(v) \leftarrow d(u) + l_{(u,v)}$

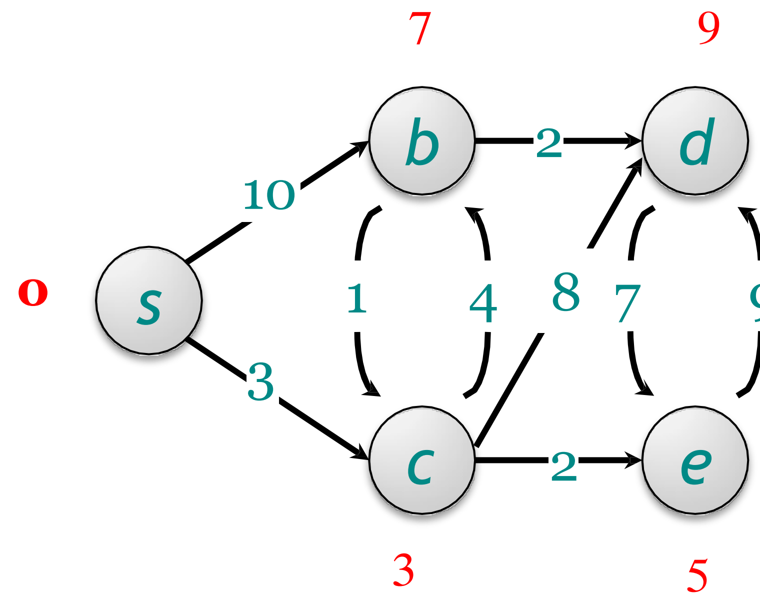


S={s, c, e, b, d}

Q={}

Example 1.

- $d(s) \leftarrow 0$
- **for each** $v \in N - \{s\}$
  - **do** $d(v) \leftarrow \infty$
- $S \leftarrow \varnothing$
- $Q \leftarrow N$
- **while** $Q \neq \varnothing$
  - **do** $u \leftarrow$ EXTRACT-MIN(Q)
    - $S \leftarrow S \cup \{u\}$
    - **for each** $v \in Adj(u)$
      - **do if** $d(v) > d(u) + l_{(u,v)}$
        - **then** $d(v) \leftarrow d(u) + l_{(u,v)}$



S={s, c, e, b, d}

Q={}

Dijkstra’s algorithm- Cnt.

- Dijkstra’s algorithm computes the shortest distances btw a start node and all other nodes in the graph (not only a target node)!
- Assumptions:
  - the graph is connected, and
  - the weights are nonnegative

Dijkstra's algorithm- Analysis

- $d(s) \leftarrow 0$
- **for** each $v \in \boldsymbol{N} - \{s\}$
  - **do** $d(v) \leftarrow \infty$
- $\boldsymbol{S} \leftarrow \varnothing$
- $\boldsymbol{Q} \leftarrow \boldsymbol{N}$
- **while** $\boldsymbol{Q} \neq \varnothing$
  - **do** $u \leftarrow \text{EXTRACT-MIN}(\boldsymbol{Q})$
    - $\boldsymbol{S} \leftarrow \boldsymbol{S} \cup \{u\}$
    - **for** each $v \in Adj(u)$
      - **do if** $d(v) > d(u) + l_{(u,v)}$
        - **then** $d(v) \leftarrow d(u) + l_{(u,v)}$

degree (u) times

$|N|$ times

Time = $\Theta(N \cdot T_{\text{EXTRACT-MIN}} + E \cdot T_{\text{Relaxation}})$, Handshaking Lemma!

Dijkstra's algorithm- Analysis- Cnt.

$$\text{Time} = \Theta\left(N \cdot T_{\text{EXTRACT-MIN}} + E \cdot T_{\text{Relaxation}}\right)$$

Q	$T_{\text{EXTRACT-MIN}}$	$T_{\text{DECREASE-KEY}}$	Total
Array	$O(N)$	$O(1)$	$O(N^2)$

Reading

- Ch.24 Single Source Shortest Paths [CLRS]
- What is Twitter, a social network or a news media? Kwak, H., et al. WWW 2010.
- Global connectivity and multilinguals in the Twitter network. Hale, S.A., SIGCHI'14.
- Fragile online relationship: a first look at unfollow dynamics in twitter. Kwak, H., et al. SIGCHI'11.
- Understanding the demographics of twitter users. Mislove, A., et al. AAAI'11