# Cascading Behavior in Networks

Advanced Social Computing

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# Lecture Topics

- **Modeling Diffusion**
- Cascades & Clusters
- Cascade Capacity

# Diffusion

- In cascades, people **imitate** behaviors of others.

- Look at cascade from network structure perspective
  - How are individuals influenced by their immediate neighbors?
    - Compatibility with technology that friends use
    - Friends political views, etc.

- “Nodes” adopt a new behavior once a **sufficient proportion of their neighbors** have done so.

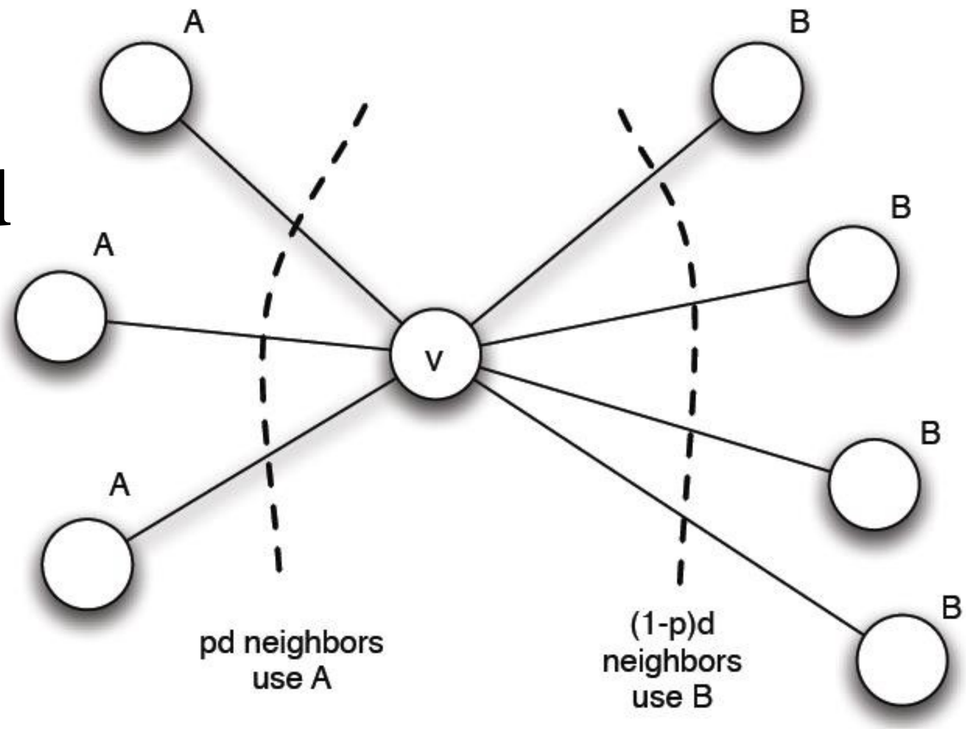
# Diffusion- Cnt.

- A Networked Coordination Game
  - Nodes choose btw two possible behaviors: A and B.
  - If nodes $v$ and $w$ are linked, then they receive payoff if their behaviors match.
    - $v$ and $w$ both adopt A, each get a payoff of **a > o**;
    - $v$ and $w$ both adopt B, each get a payoff of **b > o**;
    - $v$ and $w$ adopt opposite behaviors, each get payoff of **o**.
- Nodes choice of behavior depends on choices made by all of its neighbors, taken together!

# Diffusion- Cnt.

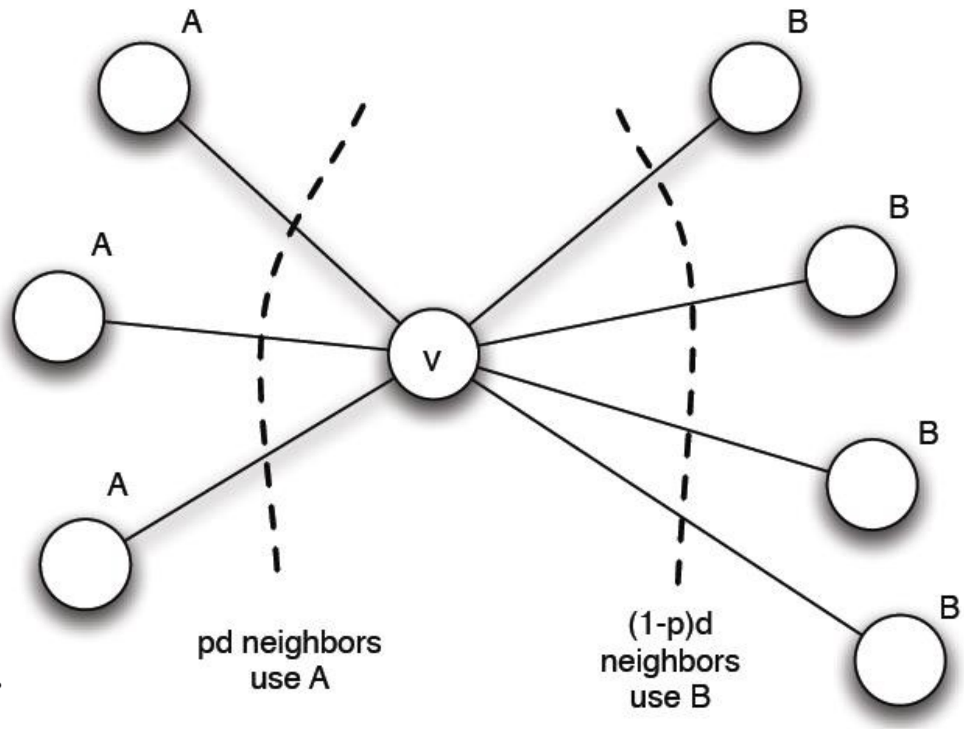
- $p$ fraction of $v$'s neighbors choose A
- $(1 - p)$ fraction choose B.
- $v$ has $d$ neighbors

- Which behavior should $v$ adopt?



# Diffusion- Cnt.

- $p$ fraction of $v$'s neighbors choose A
- $(1 - p)$ fraction choose B.
- $v$ has $d$ neighbors
  - If $v$ chooses A
    - payoff = $p \times d \times a$
  - If $v$ chooses B
    - payoff = $(1 - p) \times d \times b$
  - A is the better if




$$pda \geq (1-p)db, \qquad p \geq \frac{b}{a+b}.$$

$$\boldsymbol{q = \frac{b}{a+b}}$$

q = 0.05 vs q = 0.95

# Diffusion- Cnt.

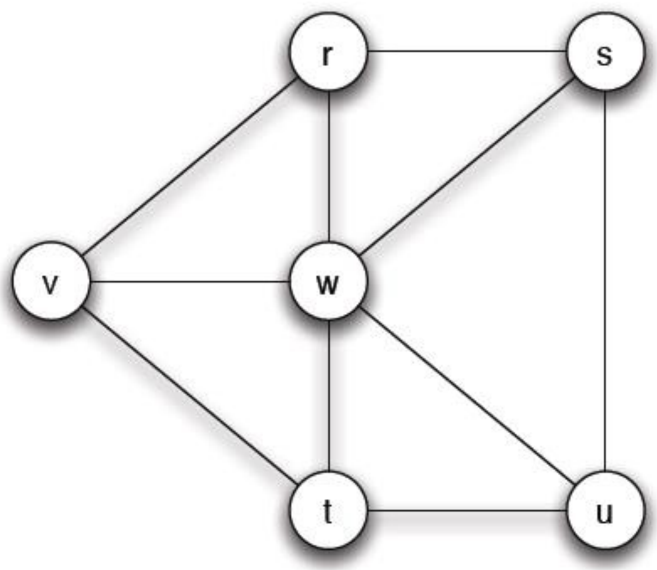
- Cascading behavior
  - Everyone adopts A,
  - Everyone adopts B,
  - **Intermediate state: some adopt A and some adopt B!**

# Diffusion- Cnt.

- Suppose everyone initially use B as a default behavior.
- A small set of **initial adopters** decide to switch to A.
- Cascade may start:
  - some neighbors of initial adopters may switch to A, then their neighbors, and so forth
- Cascade stops if:
  - **Complete cascade**: every node switch over to A!
  - We reach a step where no node wants to switch! (coexistence btw A and B)
- That depends on:
  - the network structure,
  - the choice of initial adopters,
  - the value of the threshold $q$

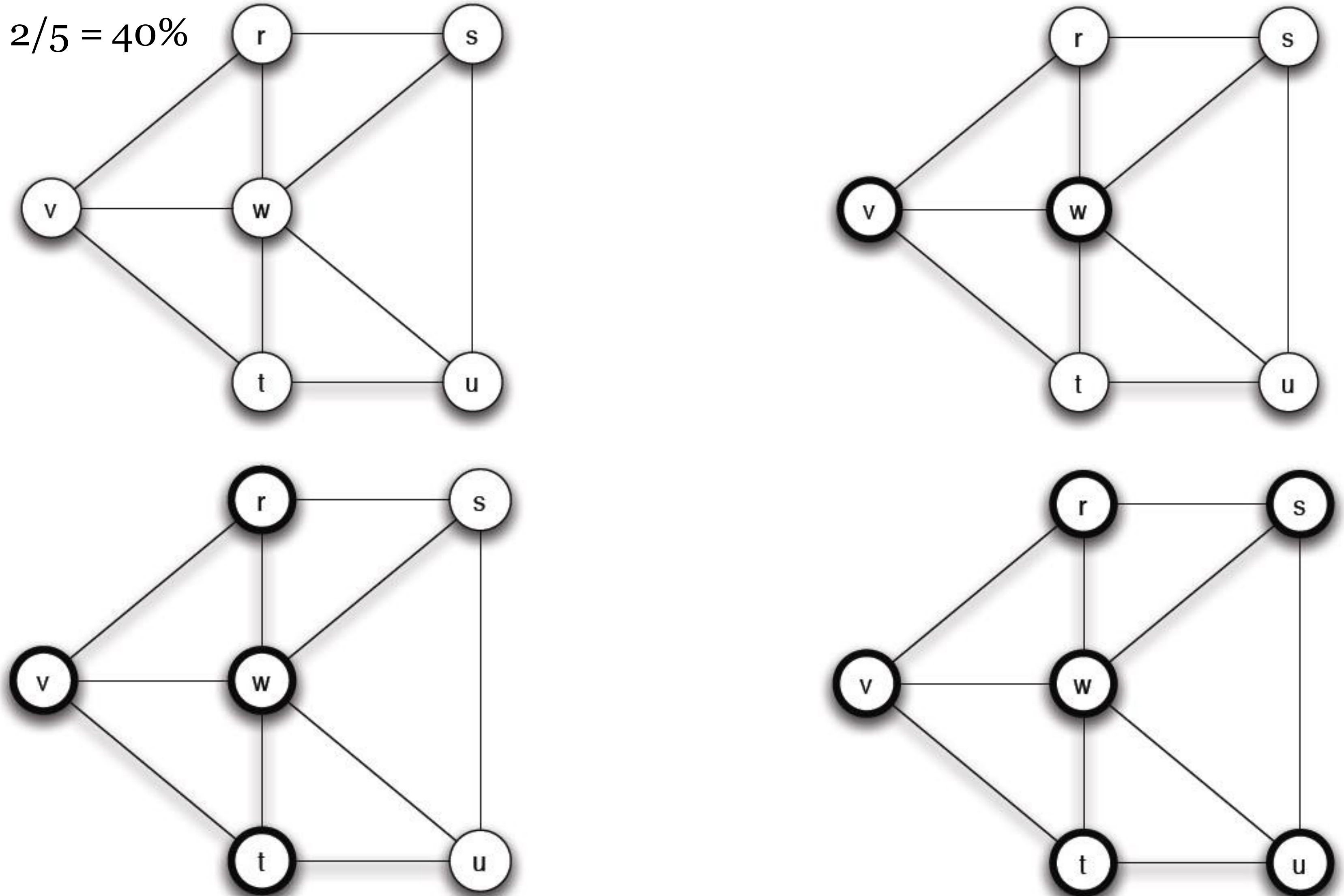
# Diffusion- Cnt.

- Payoff a=3 and b=2.
- $q$ = 2/5, nodes switch to A if at least 40% of their neighbors are using A!
- $v$ and $w$ are initial adopters of A!



# Diffusion- Cnt.

$q = 2/5 = 40\%$



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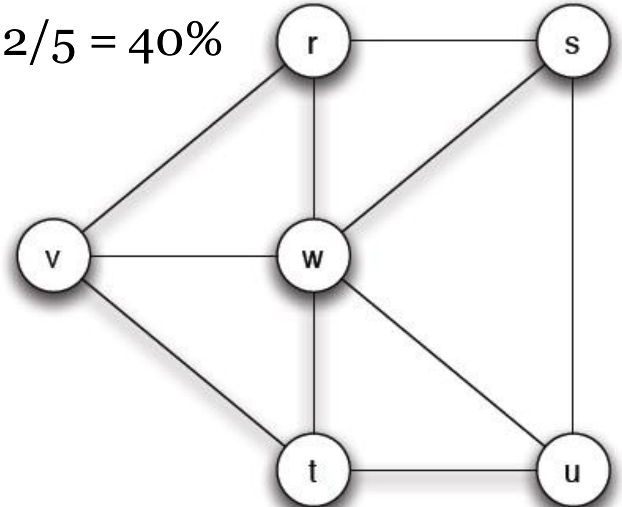
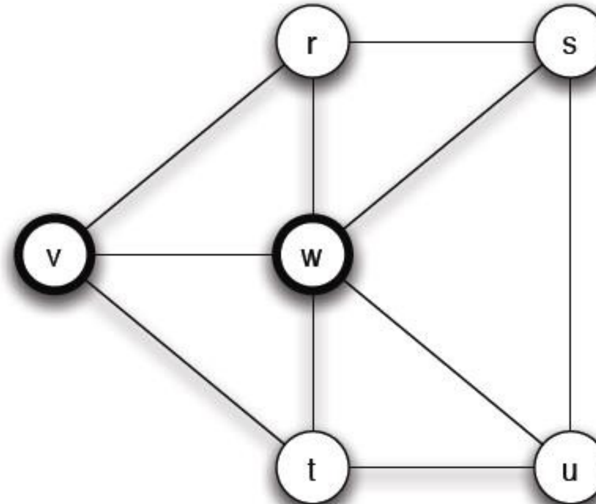
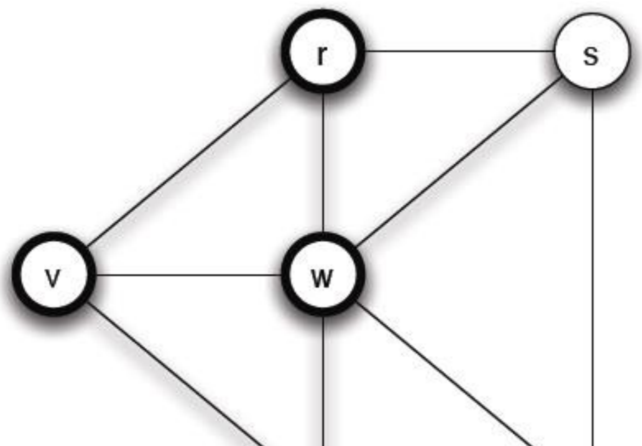
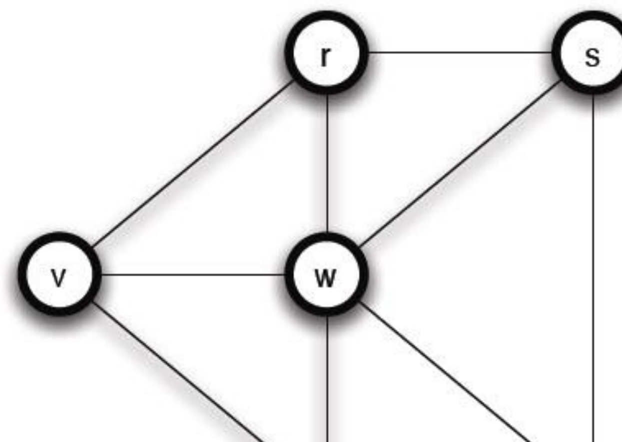


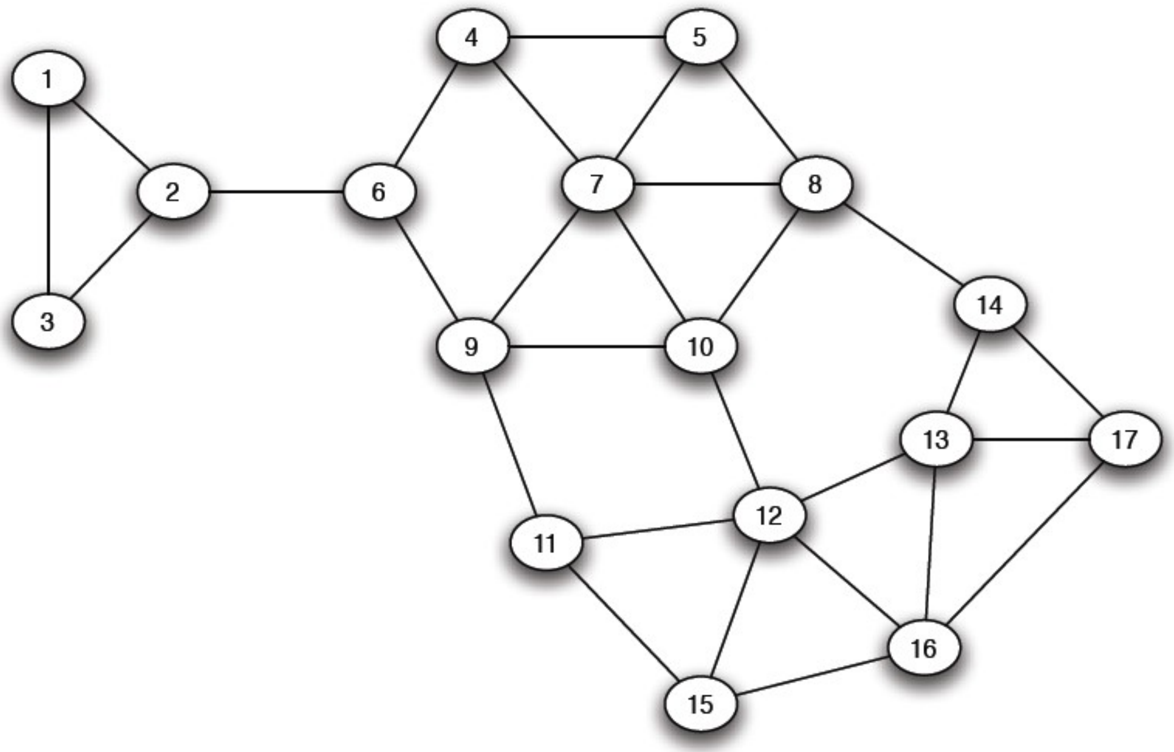




**chain reaction**: *v* and *w* aren't able to get *s* and *u* to switch by themselves, but once they've converted *r* and *t*, this provides enough leverage.

# Diffusion- Cnt.

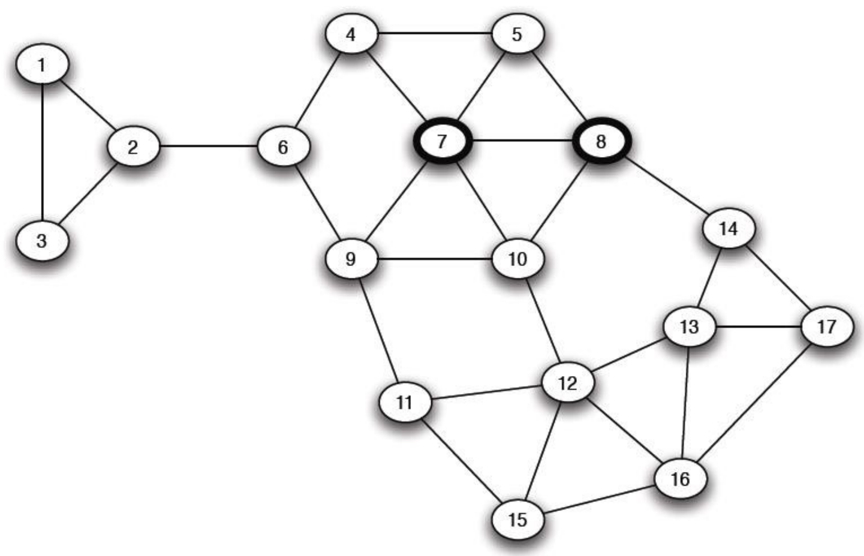
- a=3 and b=2.
- $q = 2/5$
- 7 and 8 are initial adopters of A!

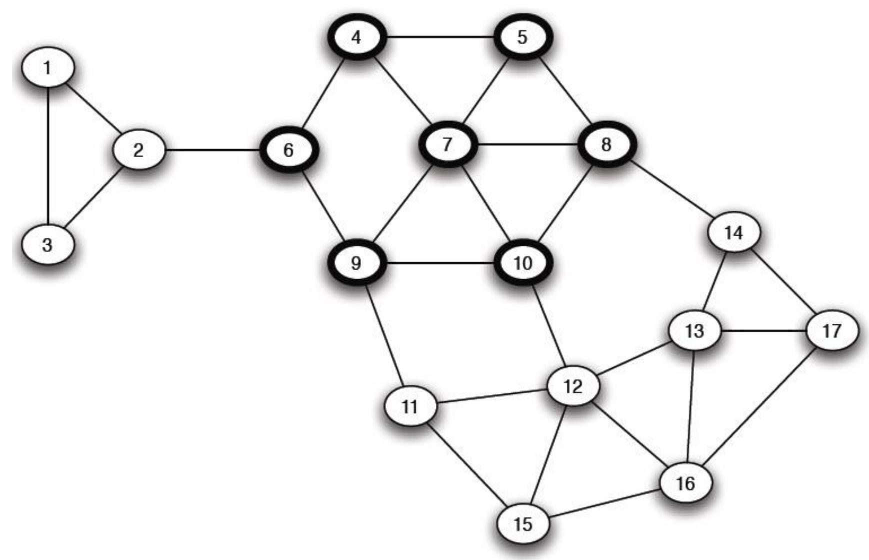


# Diffusion- Cnt.

- Takes 3 steps for the cascade to stop!
  - 5 and 10 switch to A, then
  - nodes 4 and 9, then
  - node 6.
- No further nodes will be willing to switch!

$q = 2/5$




Tightly-knit communities in the network can hinder the spread of a behavior.

# Diffusion- Cnt.

- What are useful strategies to push adoption of A (assume A and B are competing technologies)?

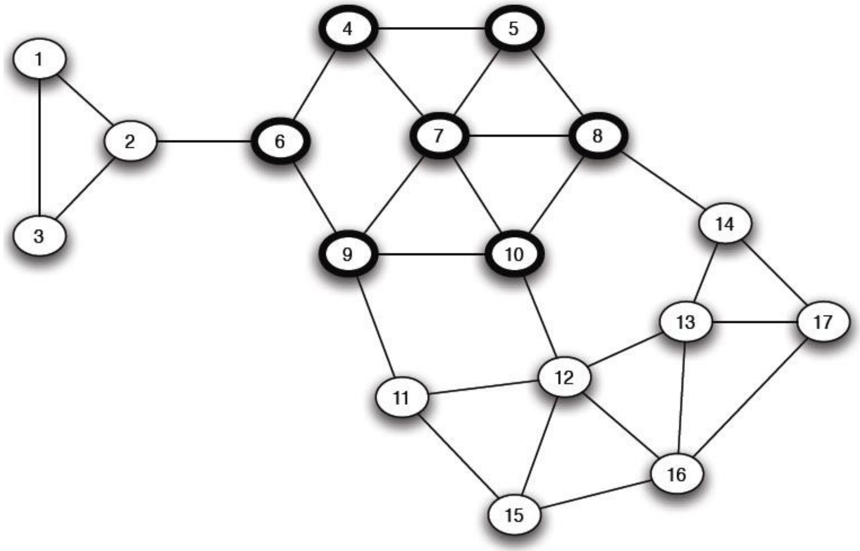
# Diffusion- Cnt.

- Strategies that are useful to push adoption of A
  - Change the payoff
    - Say from a = 3 to a = 4!
    - $q$ drops from 2/5 down to 1/3
      - then all nodes will switch to A in the above example.

$q = b/(a+b)$.

# Diffusion- Cnt.

- Strategies that are useful to push adoption of A
  - Convince a small number of key nodes in the part of the network using B to switch to A
    - Choose carefully so as to get the cascade going again!
    - Convince 12?
    - Convince 14?



# Lecture Topics

- Modeling Diffusion
- **Cascades & Clusters**
- Cascade Capacity

# Cascades & Clusters

- Question: What makes a cascade stop? Or prevents it from breaking into all parts of a network?

# Cascades & Clusters

- Question: What makes a cascade stop? Or prevents it from breaking into all parts of a network?
  - A cascade comes to stop when it runs into a **dense cluster** (tightly-knit **communities** & **homophily**),
  - This is the **only** thing that causes cascades to stop!

# Cascades & Clusters- Cnt.

- **Cluster Density**
  - A cluster with density *p* is a set of nodes such that each node has **at least** *p* fraction of its neighbors in the set.

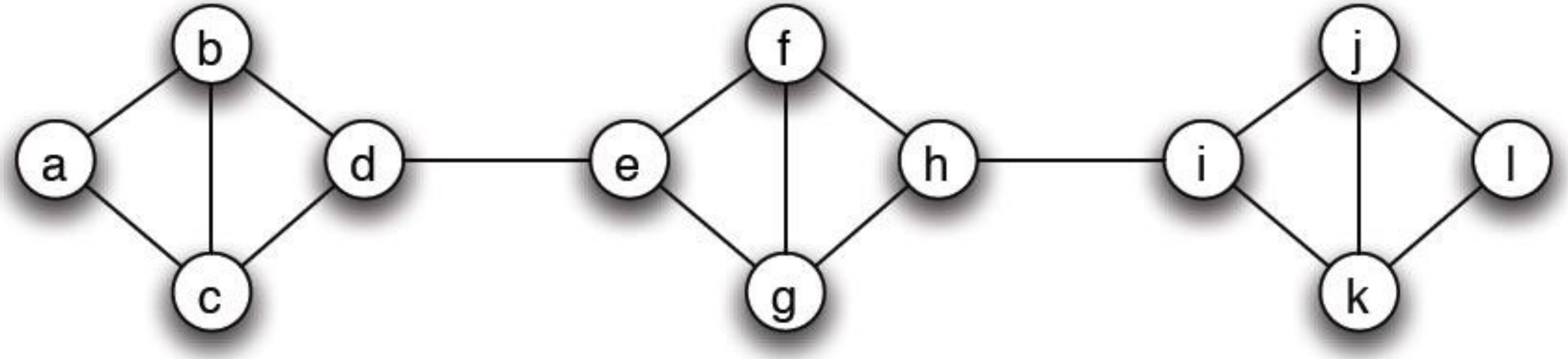
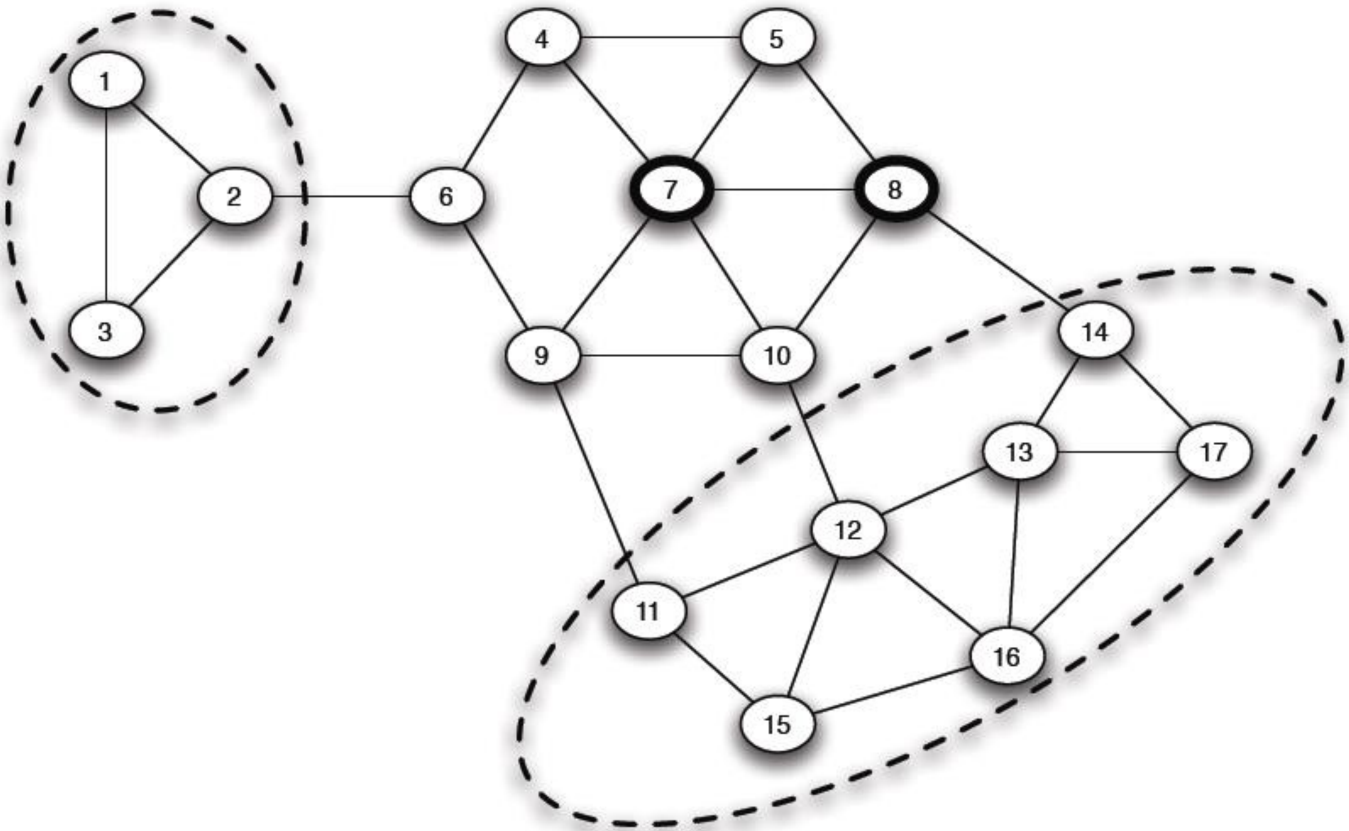



Figure 19.6: A collection of four-node clusters, each of density 2/3.

# Cascades & Clusters- Cnt.

- **Claim**: Given initial adopters of A & threshold $q$:
  i. If remaining network contains a cluster of density greater than $1 - q$, then no complete cascade.
  ii. If there is no complete cascade, the remaining network contains a cluster of density $> 1 - q$.

# Cascades & Clusters- Cnt.




$q$ = 2/5 = 40%

Cluster density = 2/3 = 66%

Cluster density > (1-$q$) = 60%

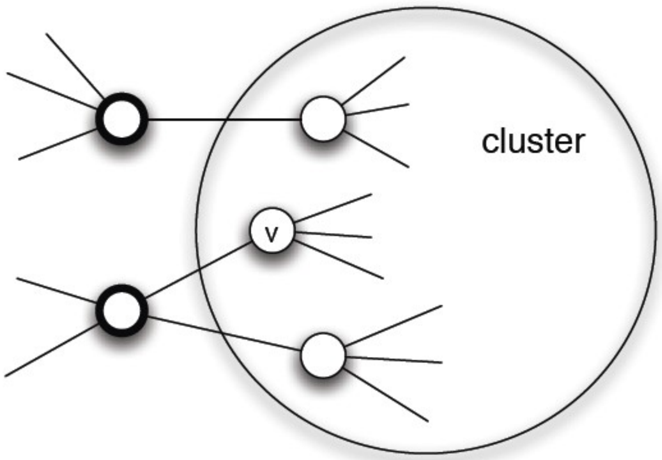
# Cascades & Clusters- Cnt.

i. If remaining network contains a cluster of density greater than $1 - q$, then no complete cascade.

# Cascades & Clusters- Cnt.

i. If remaining network contains a cluster of density greater than 1 – $q$, then no complete cascade.

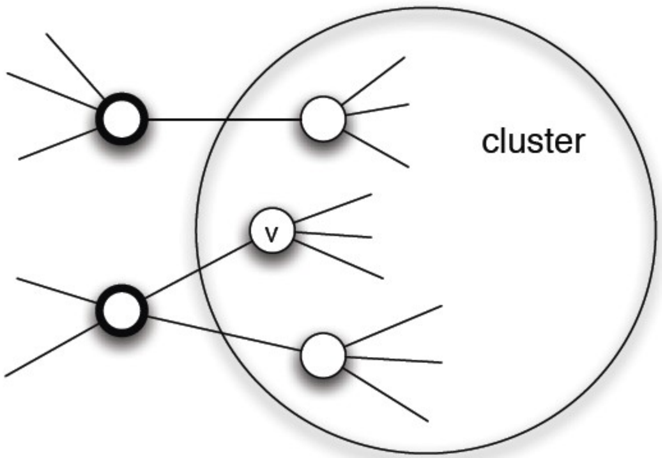
- **Solution**
  - Assume there is a node inside the cluster (density > 1-q) that adopts A
  - Let $v$ be the **first** node that does so.



# Cascades & Clusters- Cnt.

i. If remaining network contains a cluster of density greater than $1 - q$, then no complete cascade.

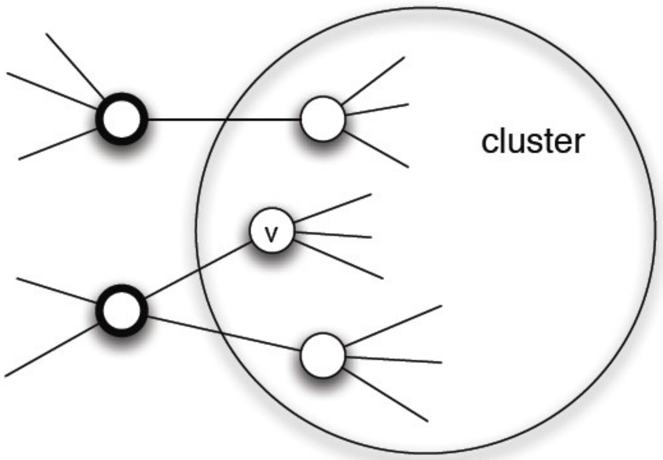
- **Solution**
  - The only neighbors of $v$ that were using A at the time it decided to switch were **outside** the cluster.



# Cascades & Clusters- Cnt.

i. If remaining network contains a cluster of density greater than $1 - q$, then no complete cascade.

- **Solution**
  - But, more than a 1-q fraction of *v*'s neighbors are inside the cluster,
  - Thus less than a *q* fraction of v's neighbors are outside the cluster.
    - Thus *v* cannot adopt A




clusters block the spread of cascades

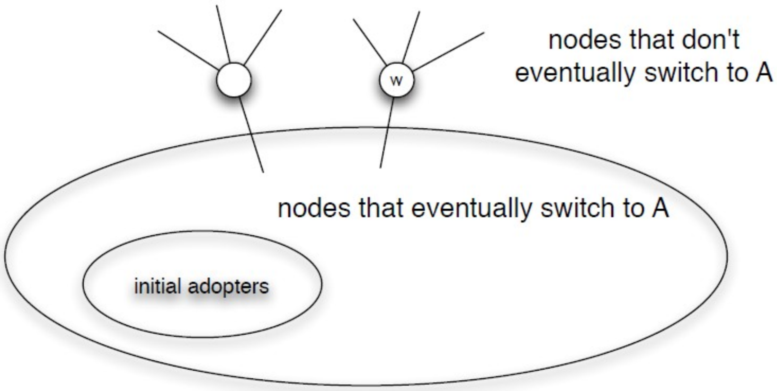
# Cascades & Clusters- Cnt.

ii. If there is no complete cascade, the remaining network contains a cluster of density $> 1 - q$.

# Cascades & Clusters- Cnt.

ii. If there is no complete cascade, the remaining network contains a cluster of density > 1 – q.

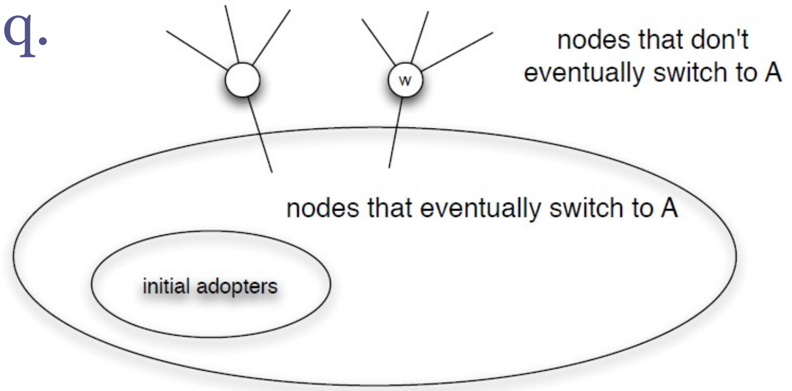
- **Solution**
  - Run the process until it stops!
    - there are nodes using B that don't want to switch.
    - let S denote such nodes.



# Cascades & Clusters- Cnt.

ii. If there is no complete cascade, the remaining network contains a cluster of density > 1 – q.

- **Solution**
  - Run the process until it stops!
    - consider any node $w \in S$
    - fraction of $w$'s neighbors using A is $< q$.
    - fraction of $w$'s neighbors using B is $> 1 - q$.
    - This holds for any node $w \in S$
      - S is a cluster of density $> 1 - q$.



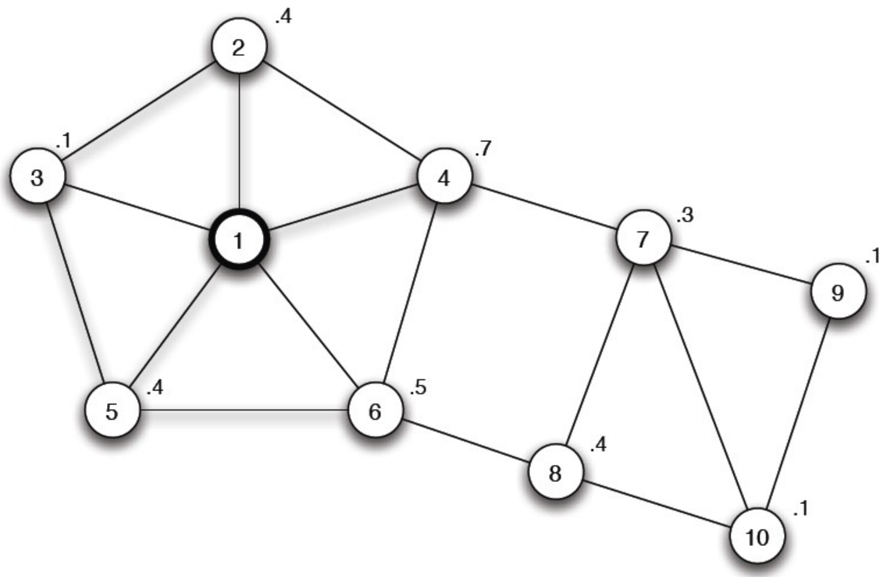

Whenever a cascade comes to a stop, there's a cluster that can be used to explain why.

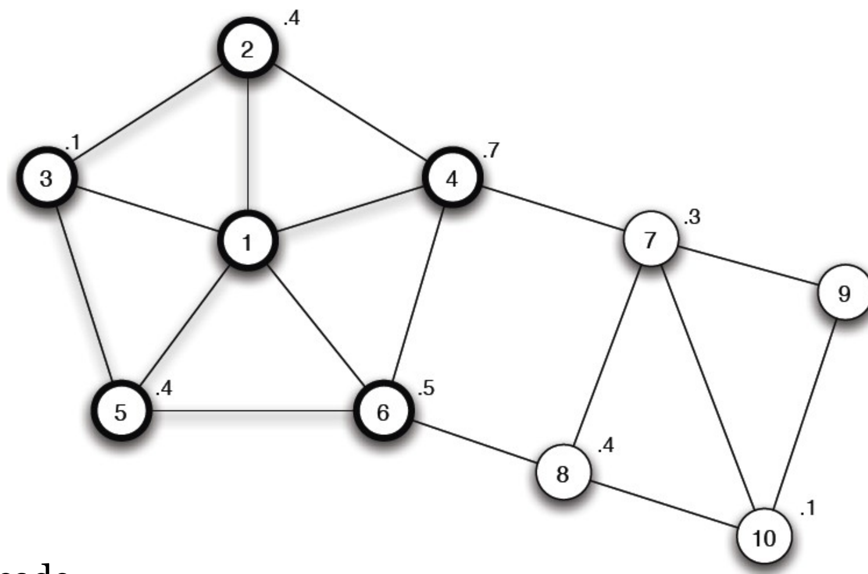
# Extensions of Cascade Model

- **Heterogeneous thresholds**
  - each nodes $v$ has a **node-specific** threshold ($q_v$) for adopting a behavior!
- $v$ has $d$ neighbors of whom a $p$ fraction have behavior A, and a (1 – p) fraction have behavior B:
  - Payoff from choosing A is $pda_v$
  - Payoff from choosing B is $(1 - p)db_v$.
- A is better for $v$ if
  - $pda_v > (1 - p)db_v$.

$$p \geq \frac{b_v}{a_v + b_v}.$$

# Extensions of Cascade Model- Cnt.




- Without node-specific thresholds, there would no cascade.
- The extremely low threshold of node 3 lead to diffusion.

> The power of **influential nodes** is correlated to the extent to which such nodes have access to easily **influenceable nodes**.

- Clusters are still obstacle to cascades
- A ***blocking cluster*** is a set of nodes for which each node $v$ has > 1-$q_v$ fraction of its neighbors in the set.
  - Heterogeneous cluster density: node-specific threshold for the fraction of friends to have in cluster.

# Lecture Topics

- Modeling Diffusion
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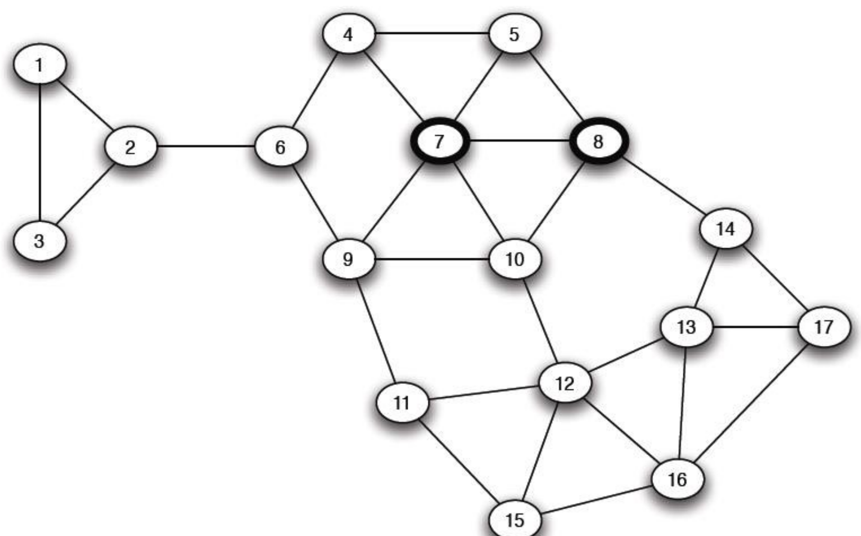
- **Cascade Capacity**

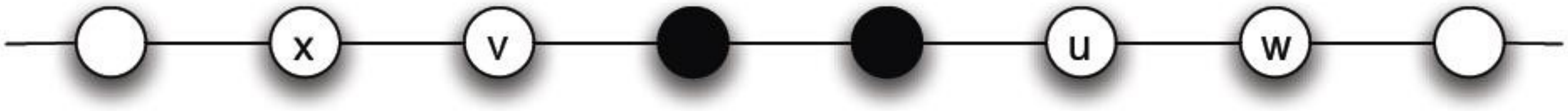
# Cascade Capacity

- **Cascade capacity** of a network: The maximum *q* for which some **small** set (***finite set***) of initial adopters can cause a **complete cascade**!
  - Indicates how different network structures are **hospitable** to cascades!



# Cascade Capacity- Cnt.

- Let S be the small set of early adopters of A.
- What is cascade capacity?
  - the maximum *q* for complete cascade?



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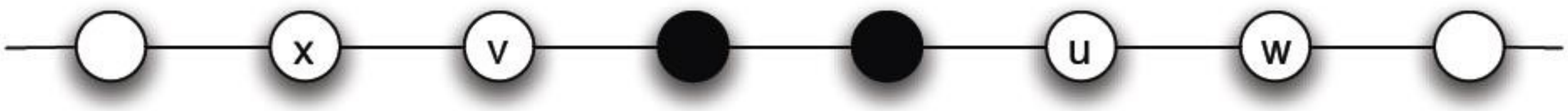
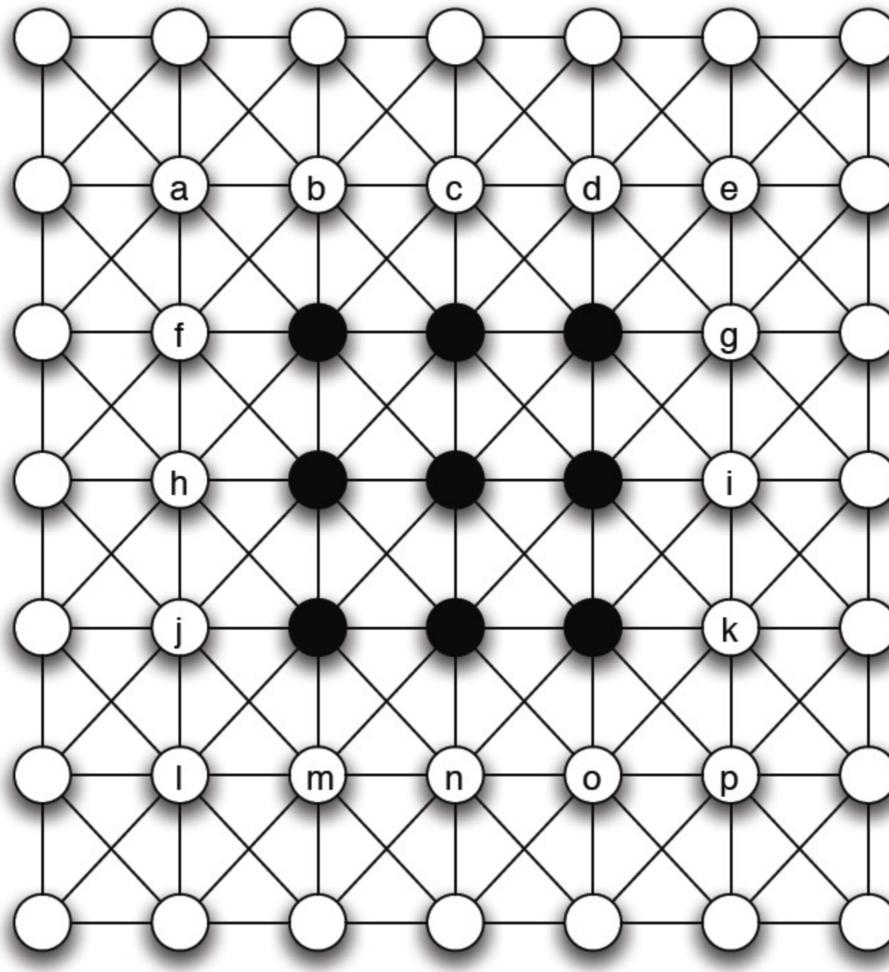



If $q <= 1/2$, complete cascade.
If $q > 1/2$, no finite set of initial adopters can get any node to switch to A.

**Cascade capacity= ½**

# Cascade Capacity- Cnt.

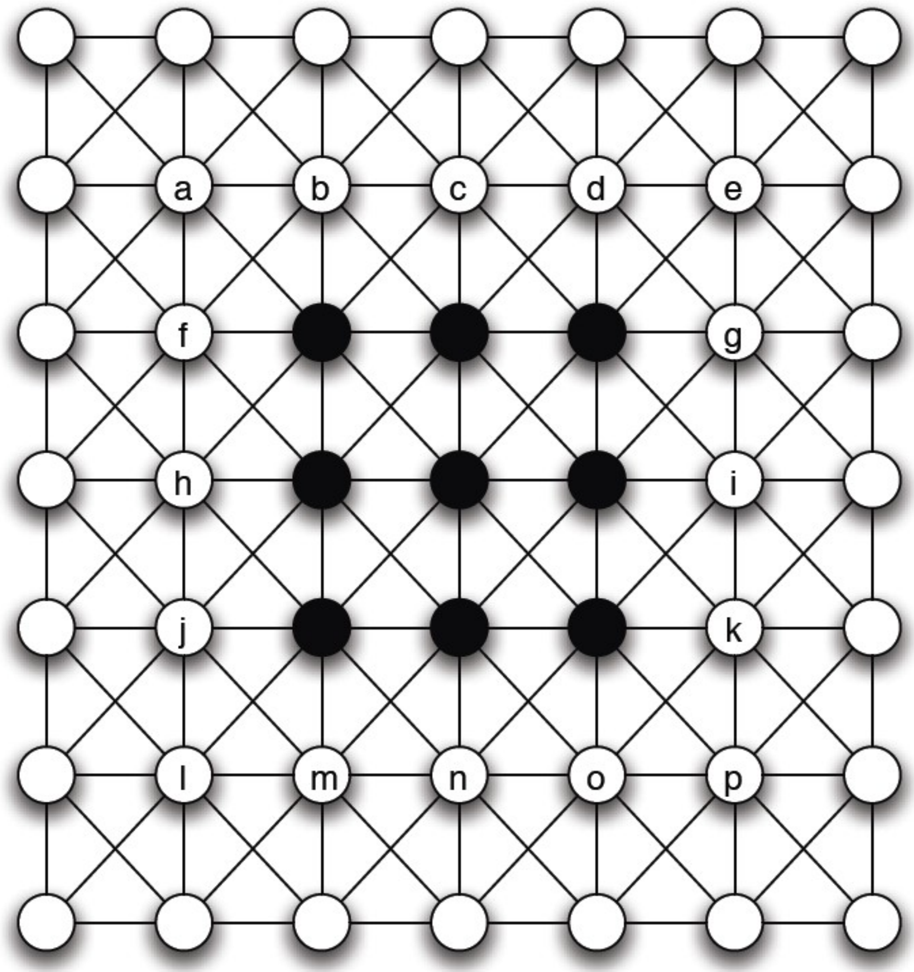


# Cascade Capacity- Cnt.

If q <= 3/8, then there is a complete cascade: first to the nodes *c*, *h*, *i*, *n*; then to nodes *b*, *d*, *f*, *g*, *j*, *k*, *m*, *o*; and then to others

If q > 3/8, no node will choose to adopt A.

**Cascade Capacity=3/8**



# Cascade Capacity- Cnt.

- How easy cascades propagate in a network with *large* cascade capacity?

# Cascade Capacity- Cnt.

- How easy cascades propagate in a network with *large* cascade capacity?
  - Cascades happen more “easily”!
  - they happen even for behaviors A that don't offer much payoff advantage over the default behavior B.

# Cascade Capacity- Cnt.

- What is the maximum possible value of cascade capacity?

# Information diffusion on Twitter

- https://snikolov.wordpress.com/2012/11/12/information-diffusion-on-twitter/

# Reading

- Ch.19 Cascading Behavior in Networks [NCM]