1 Cascading Behavior in Networks (60%)

Consider the model we studied for diffusion of a new behavior in social networks. Recall that we have a network, a behavior B that everyone starts with, and a threshold q for switching to a new behavior A; any node will switch to A if at least a q fraction of its neighbors have adopted A. Suppose that $q = 1/2$, i.e. any node will switch to A if at least half of its neighbors have adopted A. In the network of Figure 1:

1. Find a set of three nodes in the network with the property that if they act as the three initial adopters of A, then A will spread to all nodes. (In other words, find three nodes who are capable of causing a complete cascade of A.)
2. Is the set of three nodes you found in (1) the only set of three initial adopters capable of causing a cascade of A, or can you find a different set of three initial adopters who could also cause a cascade of A?
3. Find three clusters in the network, each of density greater than $1/2$, with the property that no node belongs to more than one of these clusters.
4. How does your answer to (3) help explain why there is no set consisting of only two nodes in the network that would be capable of causing a complete cascade of A?

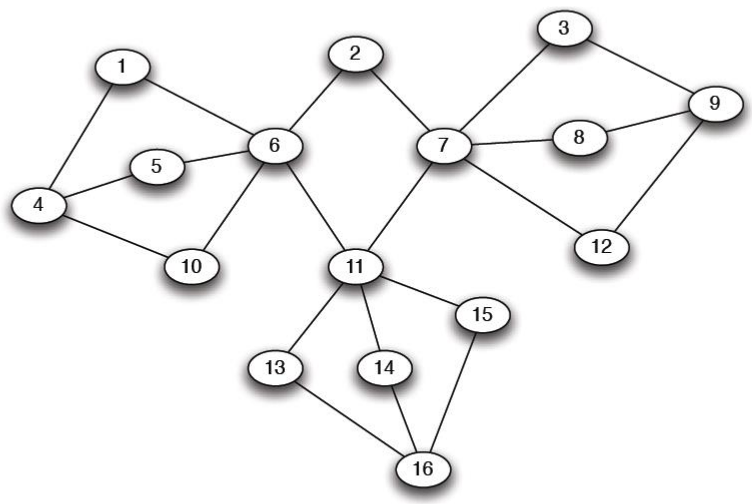

Figure 1: Network for cascading behavior analysis.

2 Six degrees of Separation (40%)

In the basic "six degrees of separation" question, one asks whether most pairs of people in the world are connected by a path of at most six edges in the social network, where an edge joins any two people who know each other on a first-name basis. Now let's consider a variation on this question. For each person in the world, we ask them to rank the 30 people they know best, in descending order of how well they know them. We then construct two different social networks:

1. The "close-friend" network: from each person we create a directed edge only to their ten closest friends on the list.

2. The "distant-friend" network: from each person we create a directed edge only to the ten people listed in positions 21 through 30 on their list.

Let's think about how the small-world phenomenon might differ in these two networks. In particular, let c be the average number of people that a person can reach in six steps in the close-friend network, and let d be the average number of people that a person can reach in six steps in the distant-friend network (taking the average over all people in the world).

When researchers have done empirical studies to compare these two types of networks, they tend to find that one of c or d is consistently larger than the other. Which of the two quantities, c or d, do you expect to be larger? Give a complete explanation for your answer.