

Cascading Behavior in Networks

Advanced Social Computing

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Lecture Topics

- **Modeling Diffusion**
- Cascades & Clusters
- Cascade Capacity
- Collective Action

Diffusion

- In cascades, people **imitate** behaviors of others.
- Look at cascade from network structure perspective
 - How are individuals influenced by their immediate neighbors?
 - Compatibility with technology that friends use
 - Friends political views, etc.
- “Nodes” adopt a new behavior once a **sufficient proportion of their neighbors** have done so.

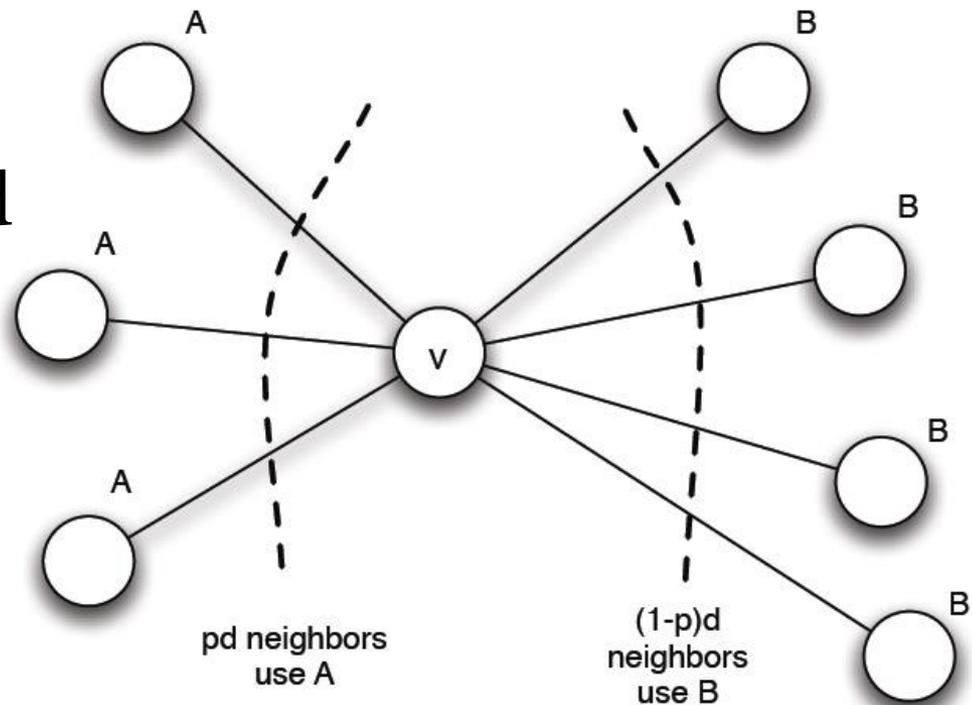
Diffusion- Cnt.

- A Networked Coordination Game
 - Nodes choose btw two possible behaviors: A and B.
 - If nodes v and w are linked, then they receive payoff if their behaviors match.
 - v and w both adopt A, each get a payoff of $\mathbf{a} > \mathbf{0}$;
 - v and w both adopt B, each get a payoff of $\mathbf{b} > \mathbf{0}$;
 - v and w adopt opposite behaviors, each get payoff of $\mathbf{0}$.
- Nodes choice of behavior depends on choices made by all of its neighbors, taken together!



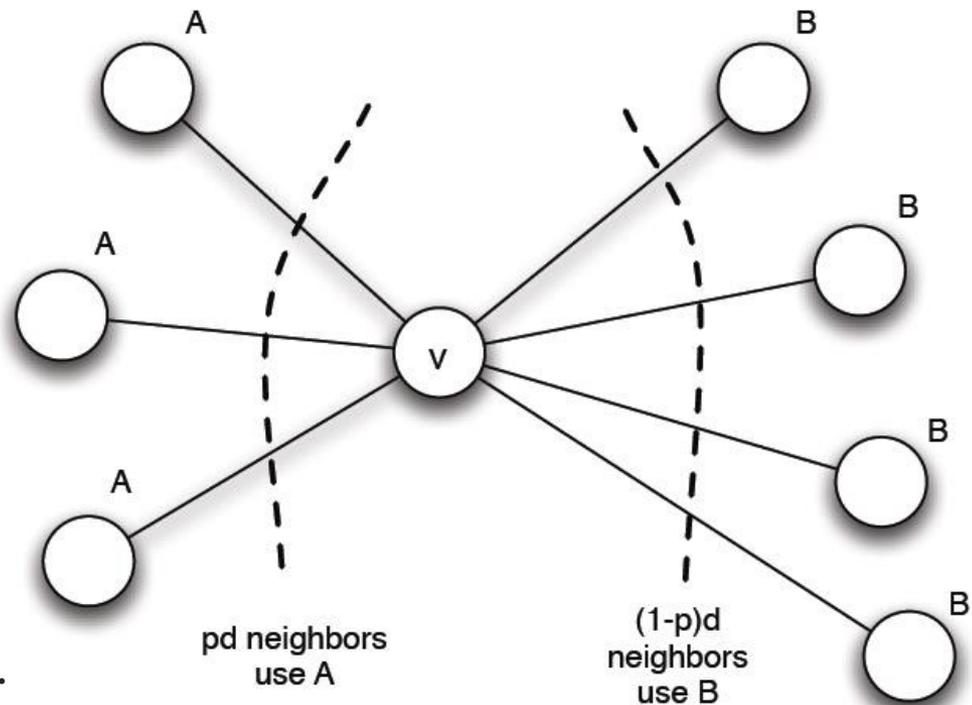
Diffusion- Cnt.

- p fraction of v 's neighbors choose A
- $(1 - p)$ fraction choose B.
- v has d neighbors
- Which behavior should v adopt?



Diffusion- Cnt.

- p fraction of v 's neighbors choose A
- $(1 - p)$ fraction choose B.
- v has d neighbors
 - If v chooses A
 - payoff = $p \times d \times a$
 - If v chooses B
 - payoff = $(1 - p) \times d \times b$



$$pda \geq (1 - p)db, \quad p \geq \frac{b}{a + b}.$$

$$q = \frac{b}{a + b}$$

$$q = 0.05 \quad \text{vs} \quad q = 0.95$$

Diffusion- Cnt.

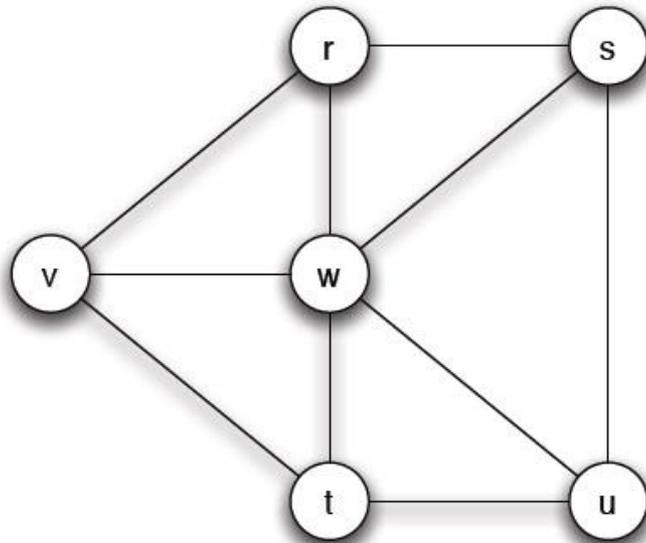
- Cascading behavior
 - Everyone adopts A,
 - Everyone adopts B,
 - **Intermediate state: some adopt A and some adopt B!**

Diffusion- Cnt.

- Suppose everyone initially use B as a default behavior.
- A small set of **initial adopters** decide to switch to A.
- Cascade may start:
 - some neighbors of initial adopters may switch to A, then their neighbors, and so forth
- Cascade stops if:
 - **Complete cascade**: every node switch over to A!
 - We reach a step where no node wants to switch!
(coexistence btw A and B)
- That depends on:
 - the network structure,
 - the choice of initial adopters,
 - the value of the threshold q

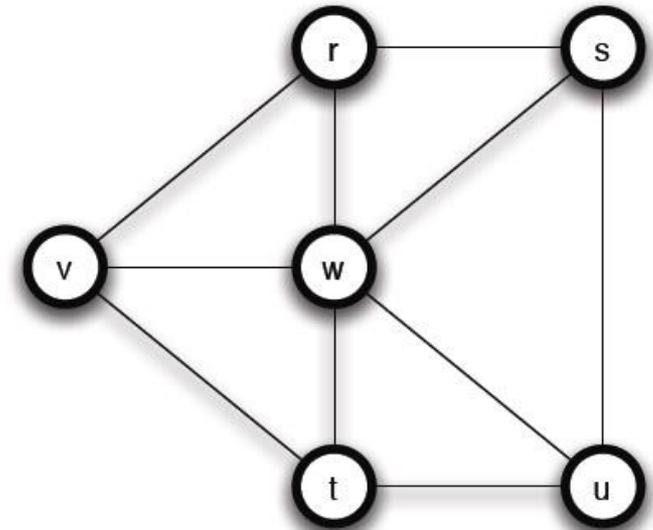
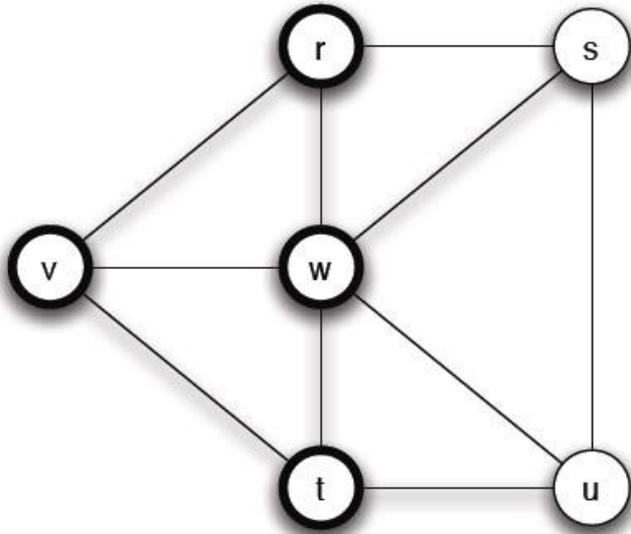
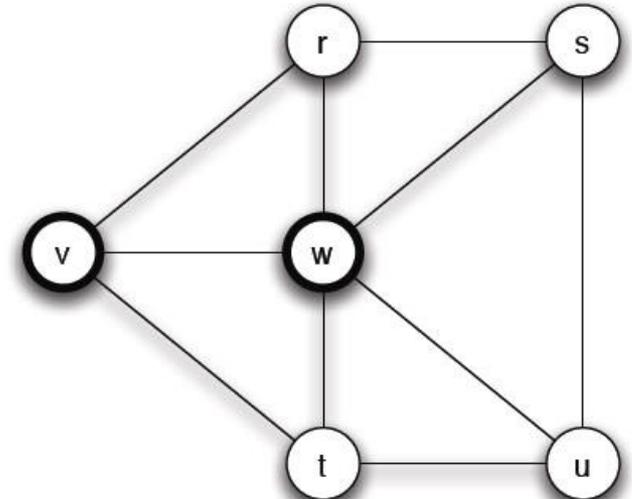
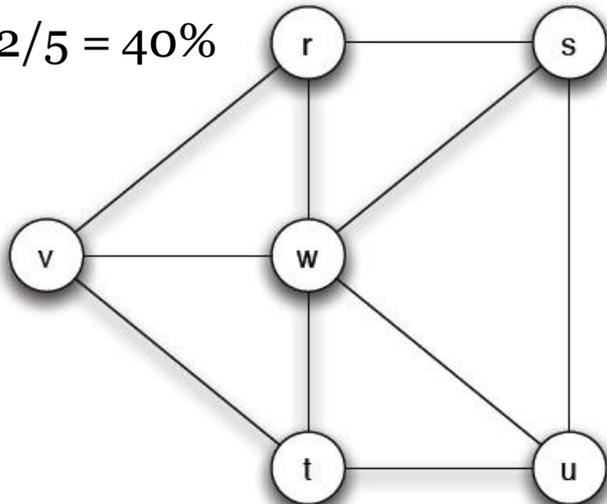
Diffusion- Cnt.

- Payoff $a=3$ and $b=2$.
- $q = 2/5$, nodes switch to A if at least 40% of their neighbors are using A!
- v and w are initial adopters of A!



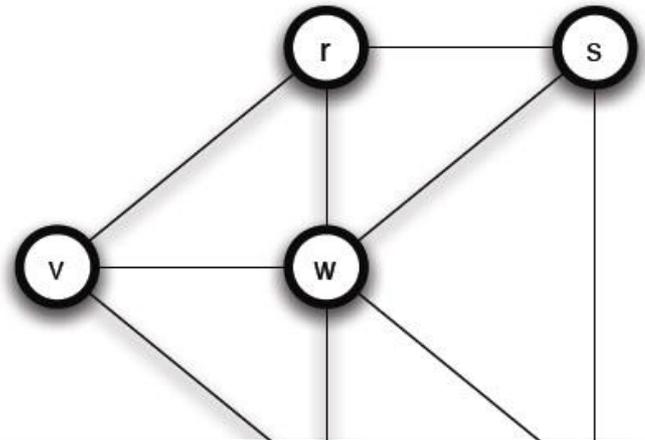
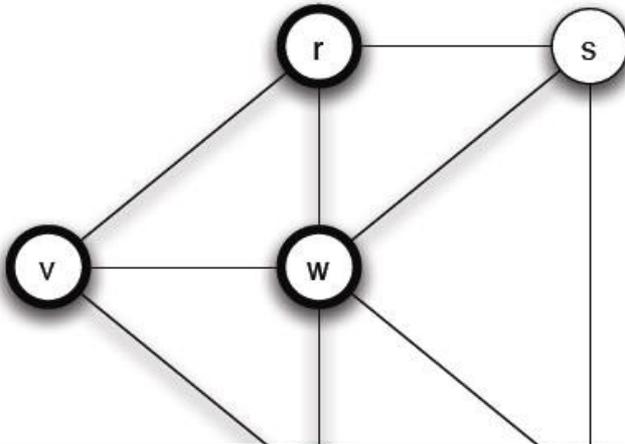
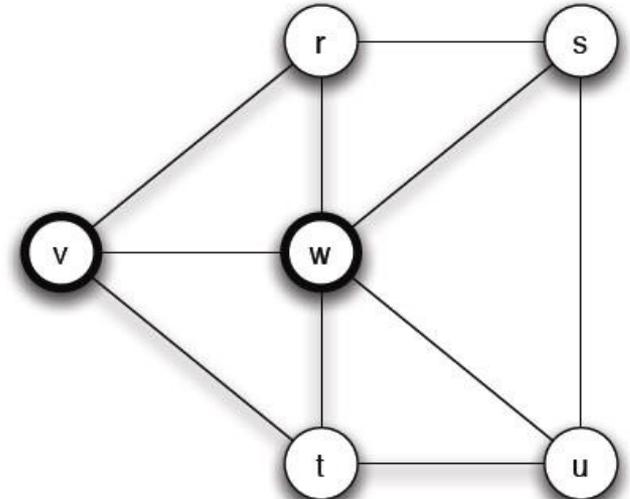
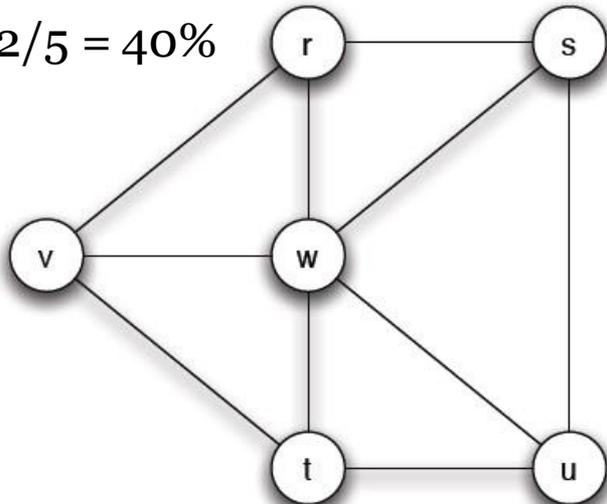
Diffusion- Cnt.

$$q = 2/5 = 40\%$$



Diffusion- Cnt.

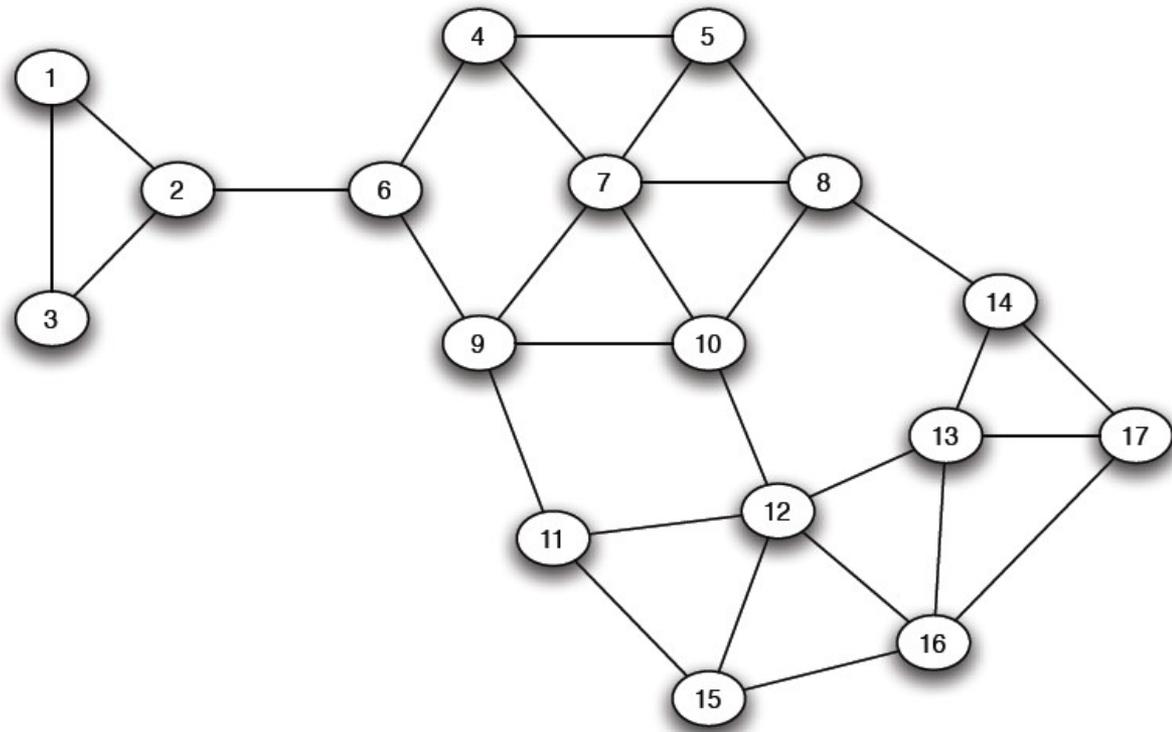
$$q = 2/5 = 40\%$$



chain reaction: *v* and *w* aren't able to get *s* and *u* to switch by themselves, but once they've converted *r* and *t*, this provides enough leverage.

Diffusion- Cnt.

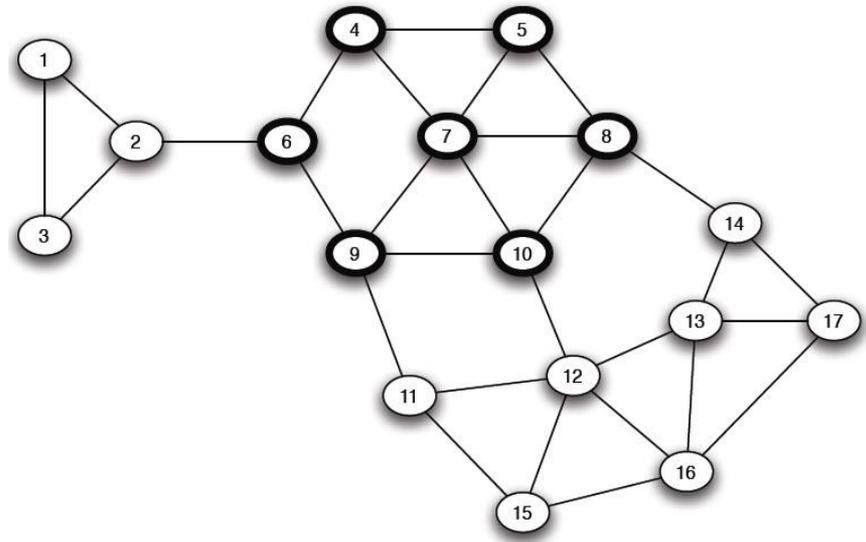
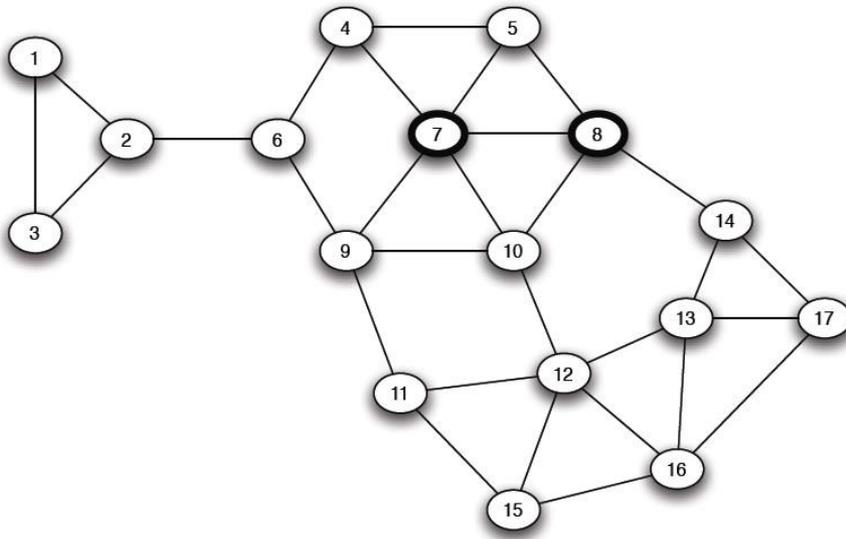
- $a=3$ and $b=2$.
- $q = 2/5$
- 7 and 8 are initial adopters of A!



Diffusion- Cnt.

- Takes 3 steps for the cascade to stop!
 - 5 and 10 switch to A, then
 - nodes 4 and 9, then
 - node 6.
- No further nodes will be willing to switch!

$$q = 2/5$$



Tightly-knit communities in the network can hinder the spread of a behavior.

Diffusion- Cnt.

- What are useful strategies to push adoption of A (assume A and B are competing technologies)?

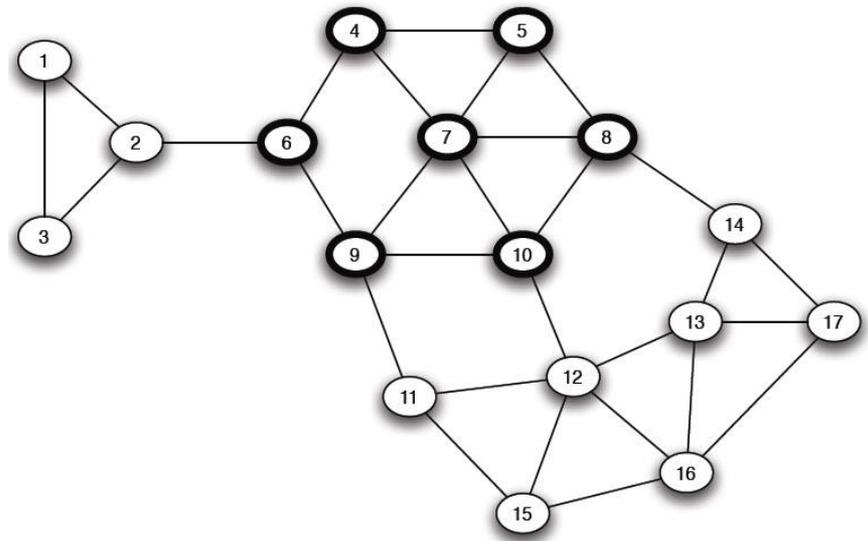
Diffusion- Cnt.

- Strategies that are useful to push adoption of A
 - Change the payoff
 - Say from $a = 3$ to $a = 4$!
 - q drops from $2/5$ down to $1/3$
 - then all nodes will switch to A in the above example.

$$q = b/(a+b).$$

Diffusion- Cnt.

- Strategies that are useful to push adoption of A
 - Convince a small number of key nodes in the part of the network using B to switch to A
 - Choose carefully so as to get the cascade going again!
 - Convince 12?
 - Convince 14?



Lecture Topics

- Modeling Diffusion
- **Cascades & Clusters**
- Cascade Capacity
- Collective Action

Cascades & Clusters

- Question: What makes a cascade stop? Or prevents it from breaking into all parts of a network?

Cascades & Clusters

- Question: What makes a cascade stop? Or prevents it from breaking into all parts of a network?
 - A cascade comes to stop when it runs into a **dense cluster** (tightly-knit **communities** & **homophily**),
 - This is the **only** thing that causes cascades to stop!

Cascades & Clusters- Cnt.

- **Cluster Density**

- A cluster with density p is a set of nodes such that each node has **at least** p fraction of its neighbors in the set.

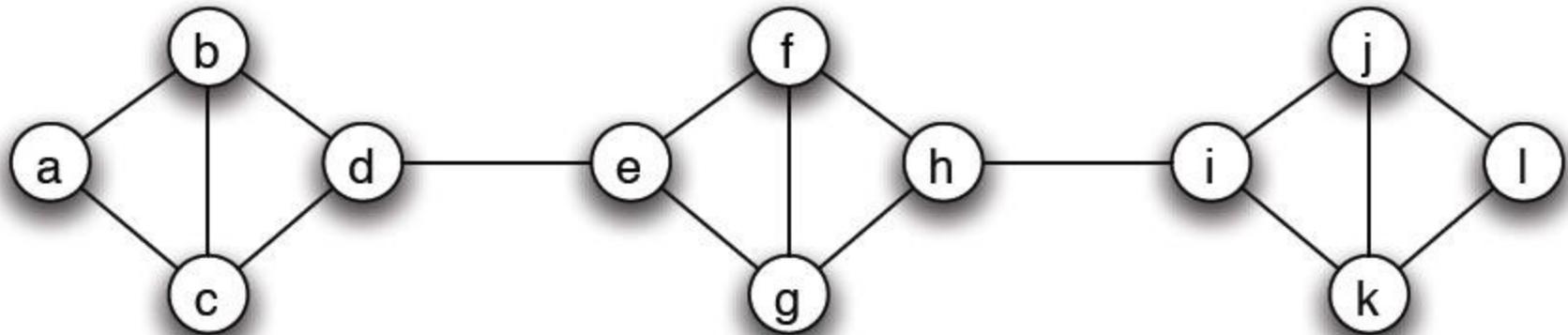
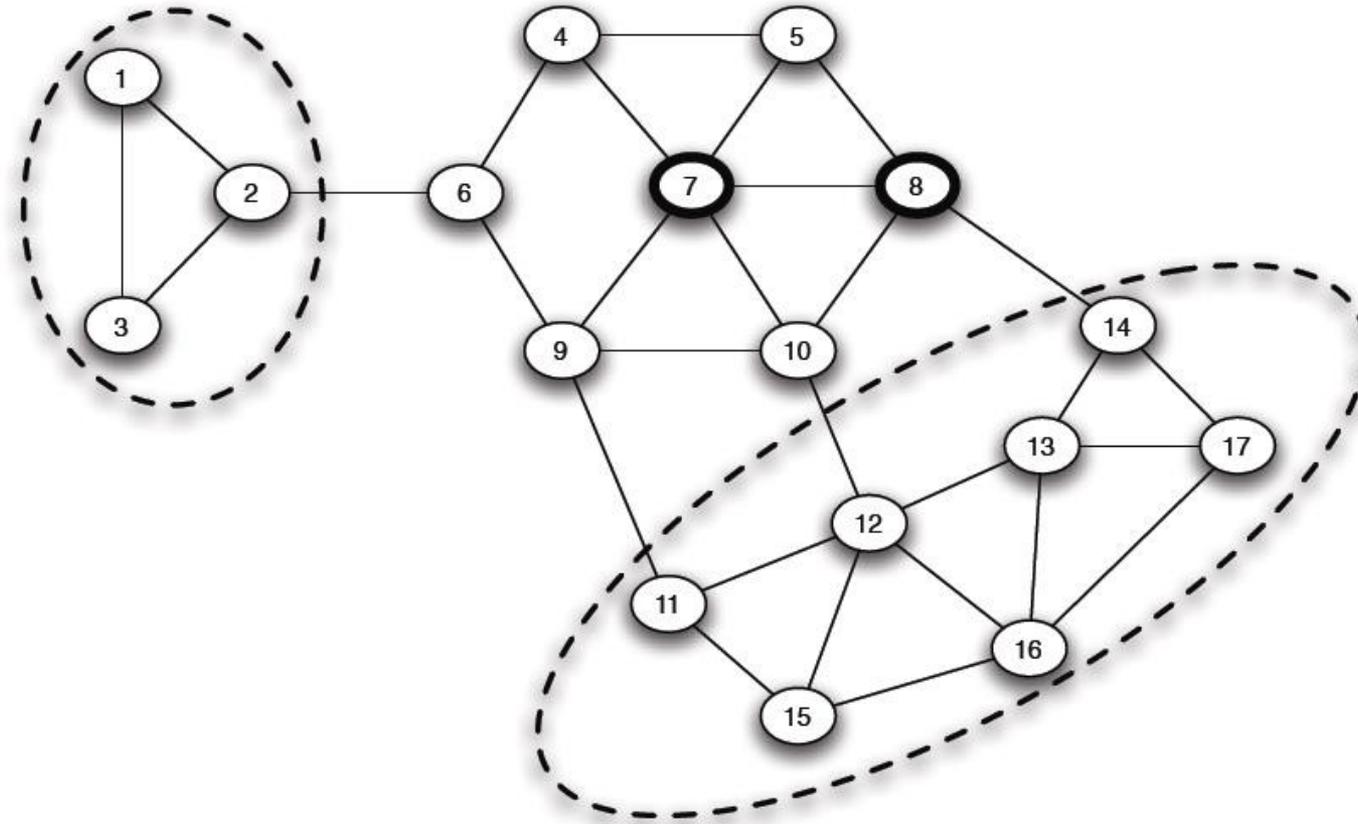


Figure 19.6: A collection of four-node clusters, each of density $2/3$.

Cascades & Clusters- Cnt.

- **Claim:** Given initial adopters of A & threshold q :
 - i. If remaining network contains a cluster of density greater than $1 - q$, then no complete cascade.
 - ii. If there is no complete cascade, the remaining network contains a cluster of density $> 1 - q$.

Cascades & Clusters- Cnt.



$$q = 2/5 = 40\%$$

$$\text{Cluster density} = 2/3 = 66\%$$

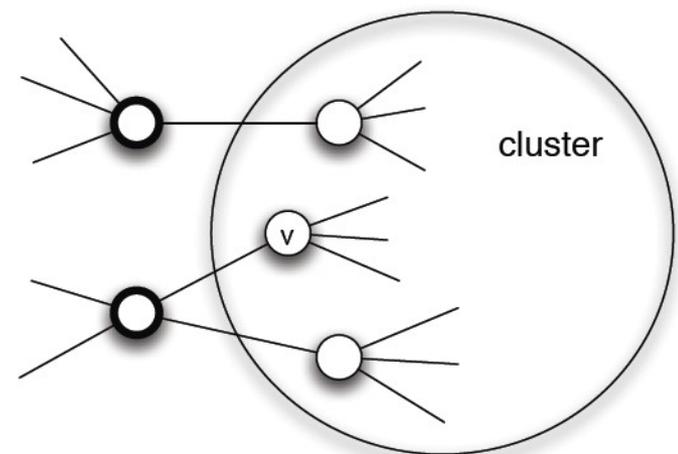
$$\text{Cluster density} > (1-q) = 60\%$$

Cascades & Clusters- Cnt.

- i. If remaining network contains a cluster of density greater than $1 - q$, then no complete cascade.

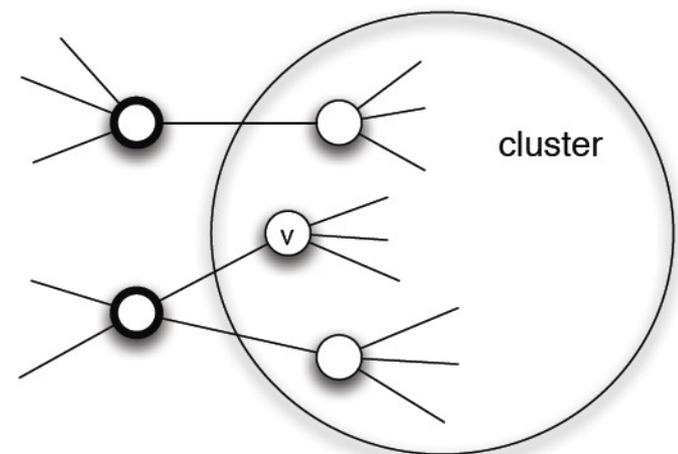
Cascades & Clusters- Cnt.

- i. If remaining network contains a cluster of density greater than $1 - q$, then no complete cascade.
- **Solution**
 - Assume there is a node inside the cluster (density $> 1 - q$) that adopts A
 - Let v be the **first** node that does so.



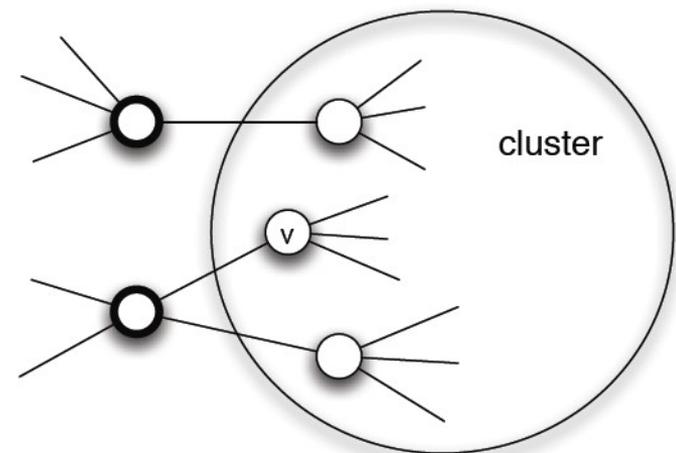
Cascades & Clusters- Cnt.

- i. If remaining network contains a cluster of density greater than $1 - q$, then no complete cascade.
- **Solution**
 - The only neighbors of v that were using A at the time it decided to switch were **outside** the cluster.



Cascades & Clusters- Cnt.

- i. If remaining network contains a cluster of density greater than $1 - q$, then no complete cascade.
- **Solution**
 - But, more than a $1-q$ fraction of v 's neighbors are inside the cluster,
 - Thus less than a q fraction of v 's neighbors are outside the cluster.
 - Thus v cannot adopt A



clusters block the spread of cascades

Cascades & Clusters- Cnt.

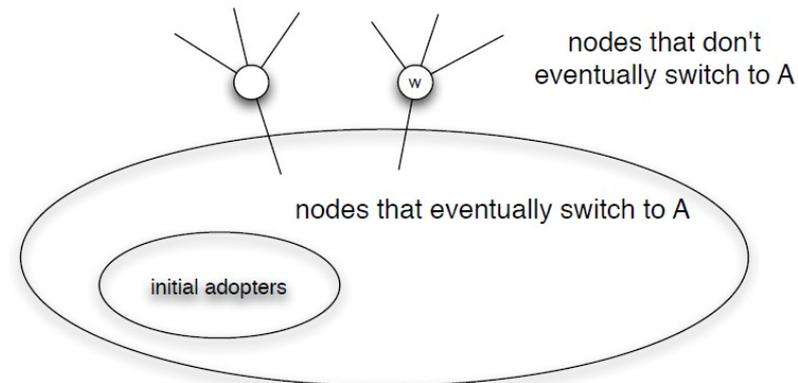
- ii. If there is no complete cascade, the remaining network contains a cluster of density $> 1 - q$.

Cascades & Clusters- Cnt.

ii. If there is no complete cascade, the remaining network contains a cluster of density $> 1 - q$.

- **Solution**

- Run the process until it stops!
 - there are nodes using B that don't want to switch.
 - let S denote such nodes.



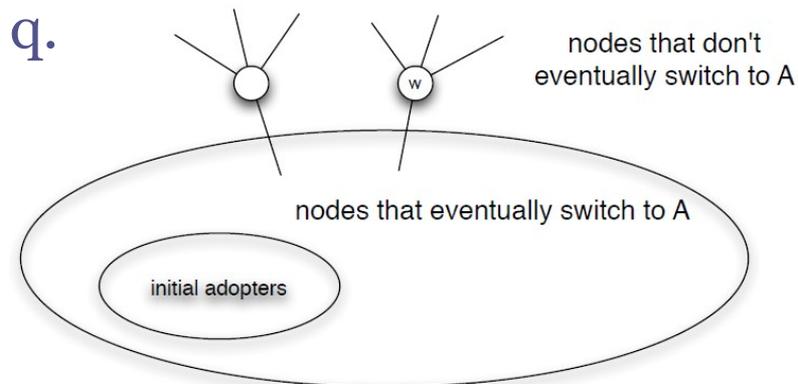
Cascades & Clusters- Cnt.

ii. If there is no complete cascade, the remaining network contains a cluster of density $> 1 - q$.

- **Solution**

- Run the process until it stops!

- consider any node $w \in S$
 - fraction of w 's neighbors using A is $< q$.
 - fraction of w 's neighbors using B is $> 1 - q$.
 - This holds for any node $w \in S$
 - S is a cluster of density $> 1 - q$.



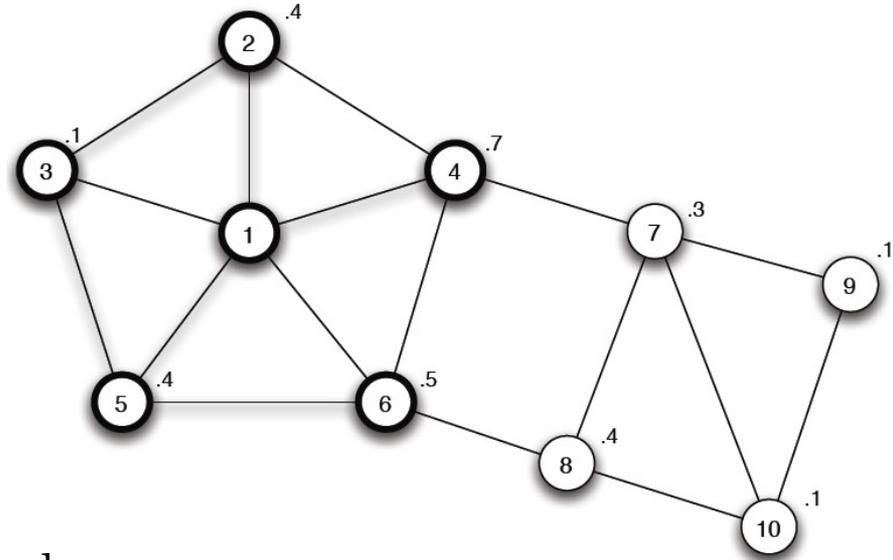
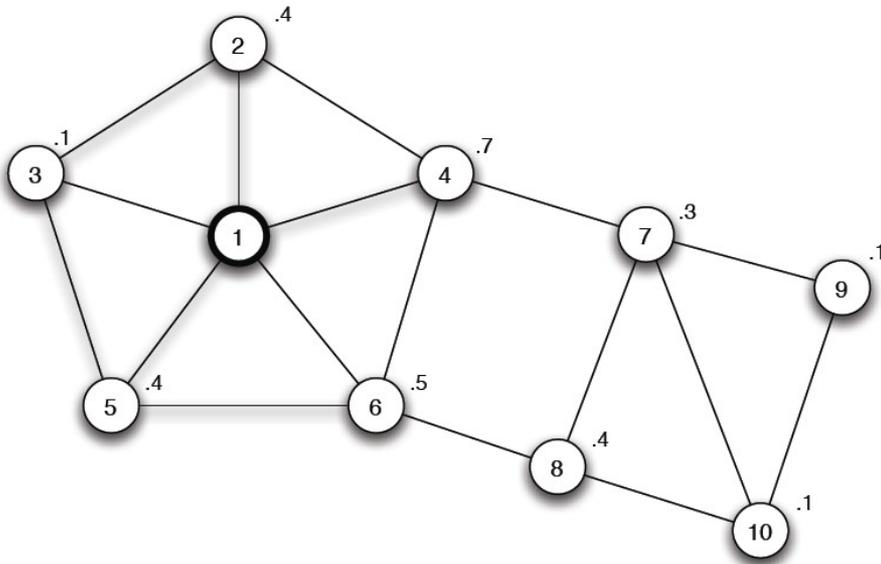
Whenever a cascade comes to a stop, there's a cluster that can be used to explain why.

Extensions of Cascade Model

- **Heterogeneous thresholds**
 - each nodes v has a **node-specific** threshold (q_v) for adopting a behavior!
- v has d neighbors of whom a p fraction have behavior A, and a $(1 - p)$ fraction have behavior B:
 - Payoff from choosing A is pda_v
 - Payoff from choosing B is $(1 - p)db_v$.
- A is better for v if
 - $pda_v > (1 - p)db_v$.

$$p \geq \frac{b_v}{a_v + b_v}.$$

Extensions of Cascade Model- Cnt.



- Without node-specific thresholds, there would no cascade.
- The extremely low threshold of node 3 lead to diffusion.

The power of **influential nodes** is correlated to the extent to which such nodes have access to easily **influenceable nodes**.

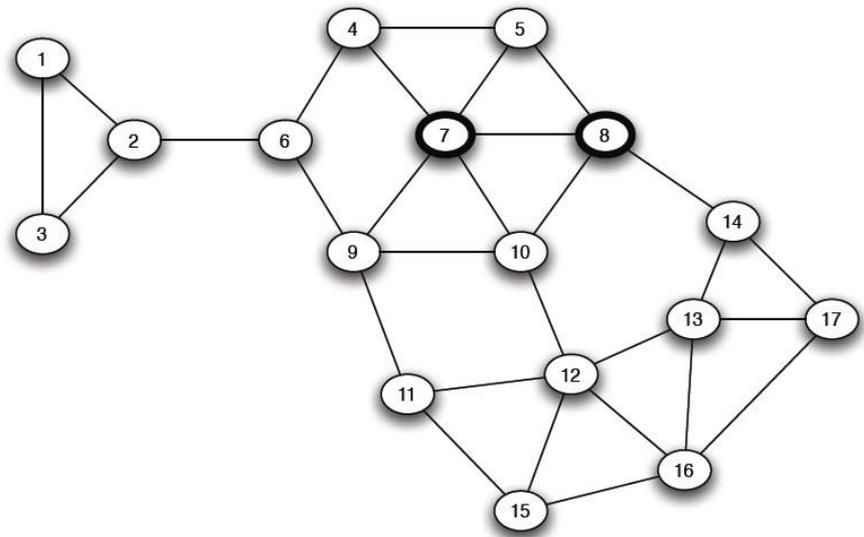
- Clusters are still obstacle to cascades
- A **blocking cluster** is a set of nodes for which each node v has $> 1 - q_v$ fraction of its neighbors in the set.
 - Heterogeneous cluster density: node-specific threshold for the fraction of friends to have in cluster.

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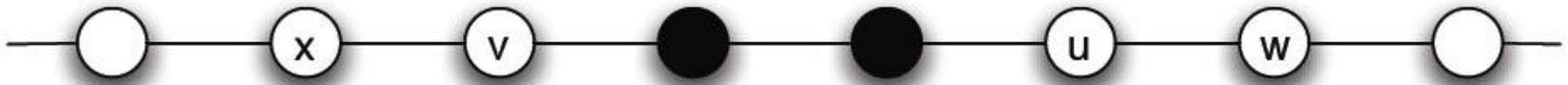
Cascade Capacity

- **Cascade capacity** of a network: The maximum q for which some **small** set (*finite set*) of initial adopters can cause a **complete cascade**!
 - Indicates how different network structures are **hospitable** to cascades!



Cascade Capacity- Cnt.

- Let S be the small set of early adopters of A .
- What is cascade capacity?
 - the maximum q for complete cascade?



Cascade Capacity- Cnt.

- Let S be the small set of early adopters of A .
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 - the maximum q for complete cascade?

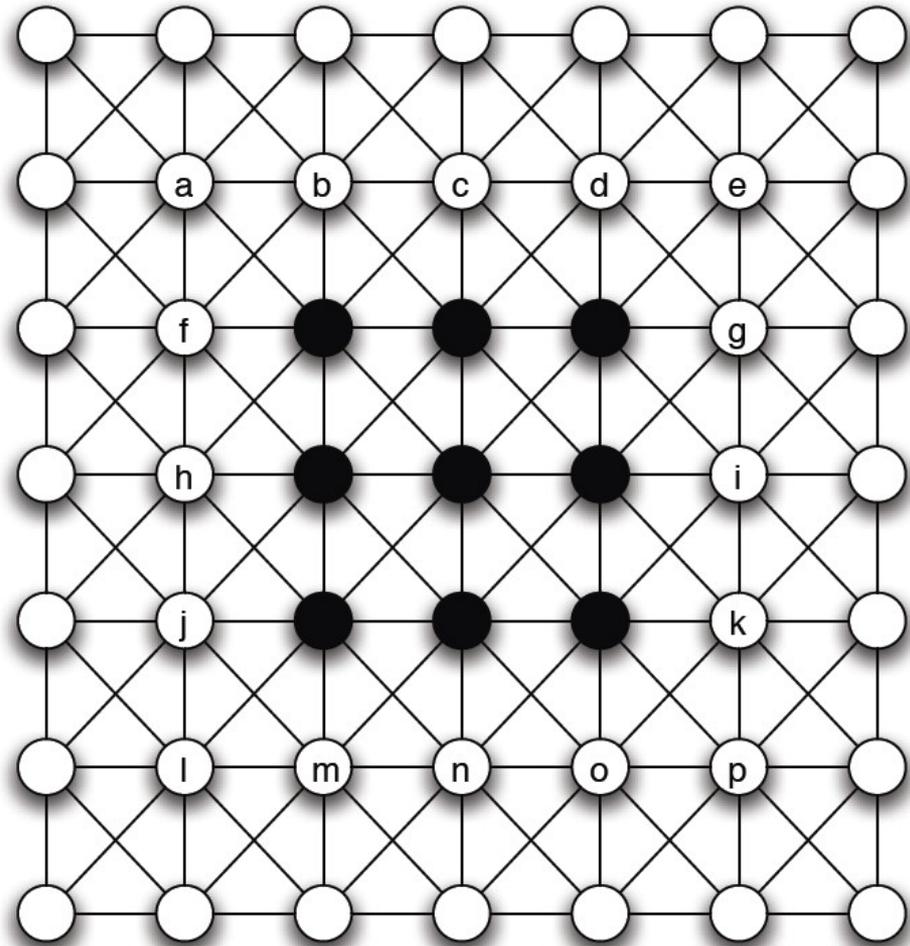


If $q \leq 1/2$, complete cascade.

If $q > 1/2$, no finite set of initial adopters can get any node to switch to A .

Cascade capacity = $1/2$

Cascade Capacity- Cnt.

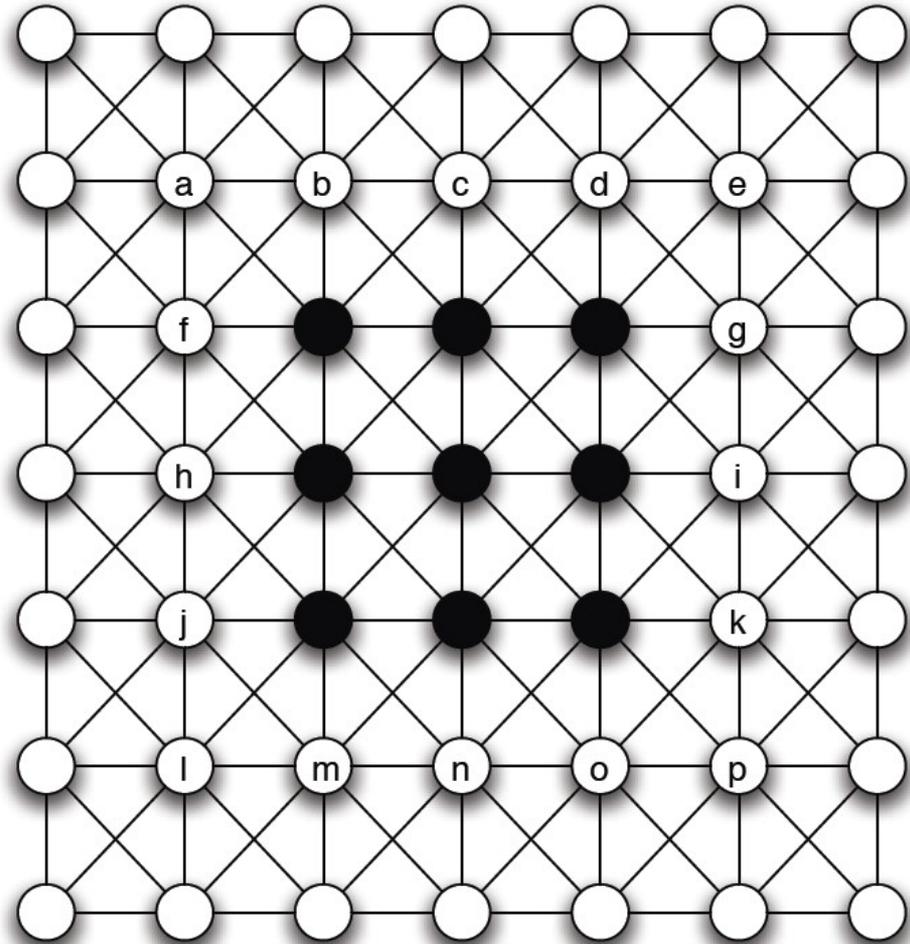


Cascade Capacity- Cnt.

If $q \leq 3/8$, then there is a complete cascade: first to the nodes c, h, i, n ; then to nodes b, d, f, g, j, k, m, o ; and then to others

If $q > 3/8$, no node will choose to adopt A.

Cascade Capacity=3/8



Cascade Capacity- Cnt.

- How easy cascades propagate in a network with *large* cascade capacity?

Cascade Capacity- Cnt.

- How easy cascades propagate in a network with *large* cascade capacity?
 - Cascades happen more “easily”!
 - they happen even for behaviors A that don't offer much payoff advantage over the default behavior B.

Cascade Capacity- Cnt.

- What is the maximum possible value of cascade capacity?

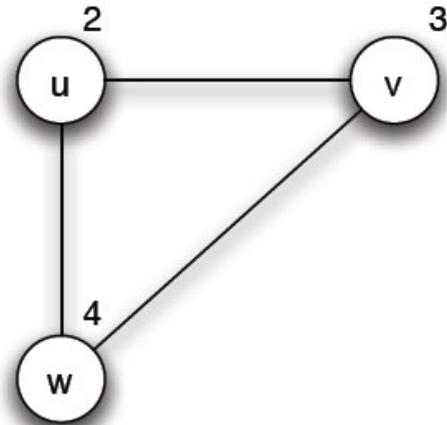
Lecture Topics

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- **Collective Action**

Collective Action

- Collective action
 - Consider a protest against something that we are good at complaining about 😊
 - Each node v shows up if at least v_k people (including itself) show up.
 - Each person knows the thresholds of her neighbors and structure of the net.
- Given a network with a set of thresholds, is the protest likely to happen?

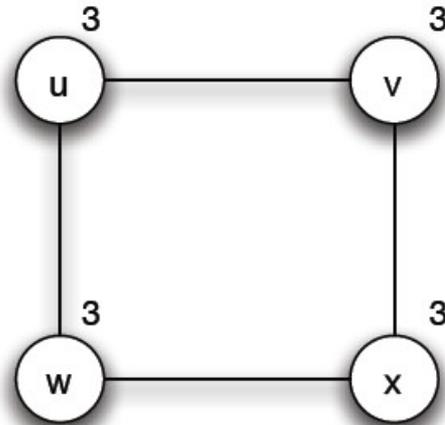
Collective Action- Cnt.



(a) *An uprising will not occur*

- w never join
- v knows w 's threshold, so v knows that w won't participate.
- v 's threshold is 3, v won't participate either.

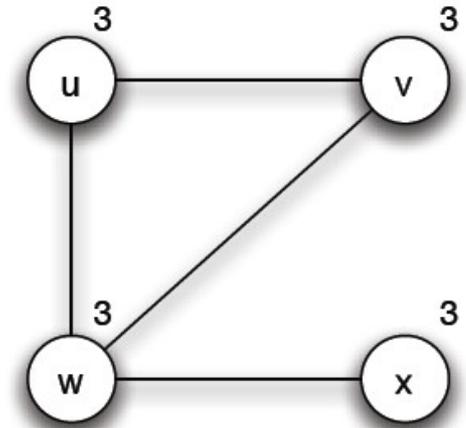
Collective Action- Cnt.



(b) *An uprising will not occur*

- u doesn't know x 's threshold
- x 's threshold could be high, like 5. $\rightarrow v$ and w don't join \rightarrow if u join, she'd be the only one.
- Each node knows there are 3 nodes with thresholds 3 (enough to form protest) but they hold back as they are not sure that other nodes know this.

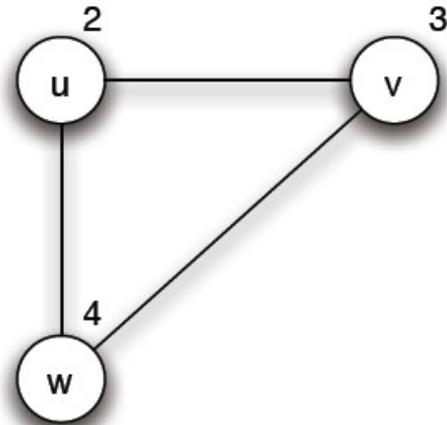
Collective Action- Cnt.



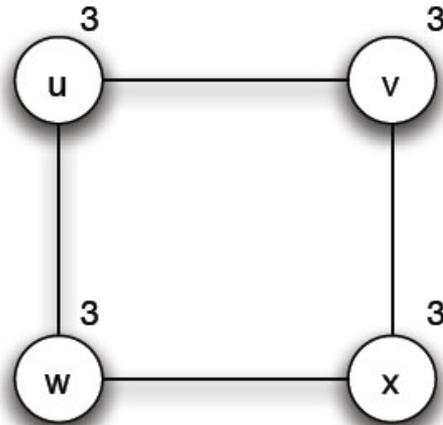
(c) *An uprising can occur*

- each of u , v , and w knows that there are 3 nodes with thresholds of 3
- And this fact is “common knowledge”

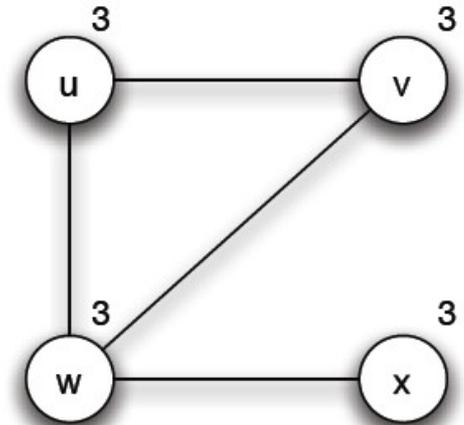
Collective Action- Cnt.



(a) *An uprising will not occur*



(b) *An uprising will not occur*



(c) *An uprising can occur*

Social nets allow individuals to base decisions on what others know.

Information diffusion on Twitter

- <https://snikolov.wordpress.com/2012/11/12/information-diffusion-on-twitter/>

Reading

- Ch.19 Cascading Behavior in Networks [NCM]