

# Cascading Behavior in Networks

Advanced Social Computing

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# Lecture Topics

- **Modeling Diffusion**
- Cascades & Clusters
- Cascade Capacity
- Collective Action

# Diffusion

- In cascades, people **imitate** behaviors of others.
- Look at cascade from network structure perspective
  - How are individuals influenced by their immediate neighbors?
    - Compatibility with technology that friends use
    - Friends political views, etc.
- “Nodes” adopt a new behavior once a **sufficient proportion of their neighbors** have done so.

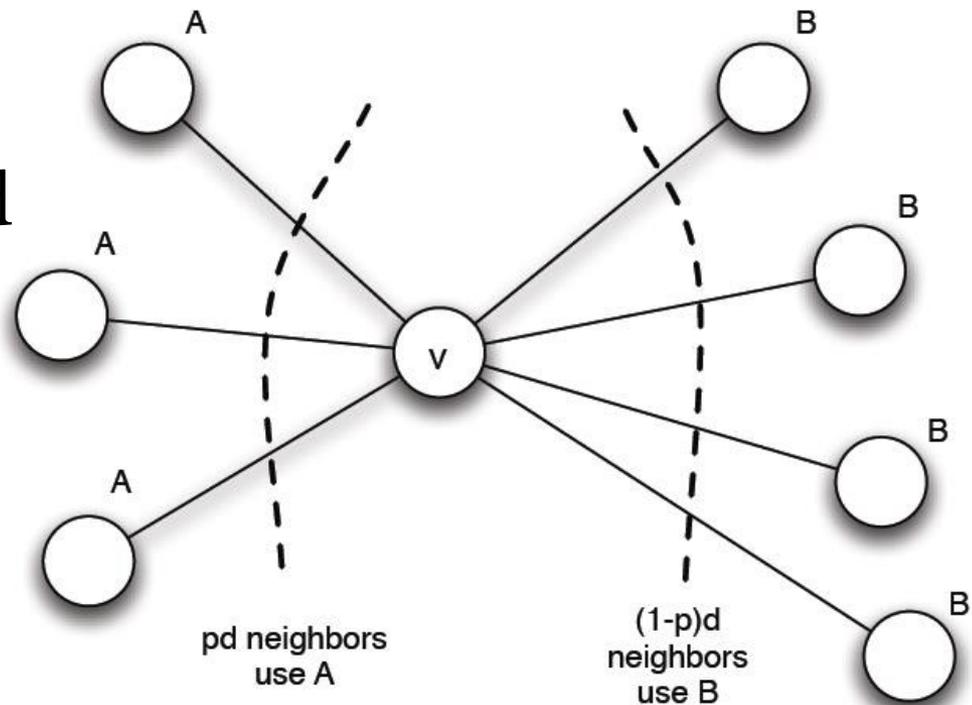
# Diffusion- Cnt.

- A Networked Coordination Game
  - Nodes choose btw two possible behaviors: A and B.
  - If nodes  $v$  and  $w$  are linked, then they receive payoff if their behaviors match.
    - $v$  and  $w$  both adopt A, each get a payoff of  $\mathbf{a} > \mathbf{0}$ ;
    - $v$  and  $w$  both adopt B, each get a payoff of  $\mathbf{b} > \mathbf{0}$ ;
    - $v$  and  $w$  adopt opposite behaviors, each get payoff of  $\mathbf{0}$ .
- Nodes choice of behavior depends on choices made by all of its neighbors, taken together!



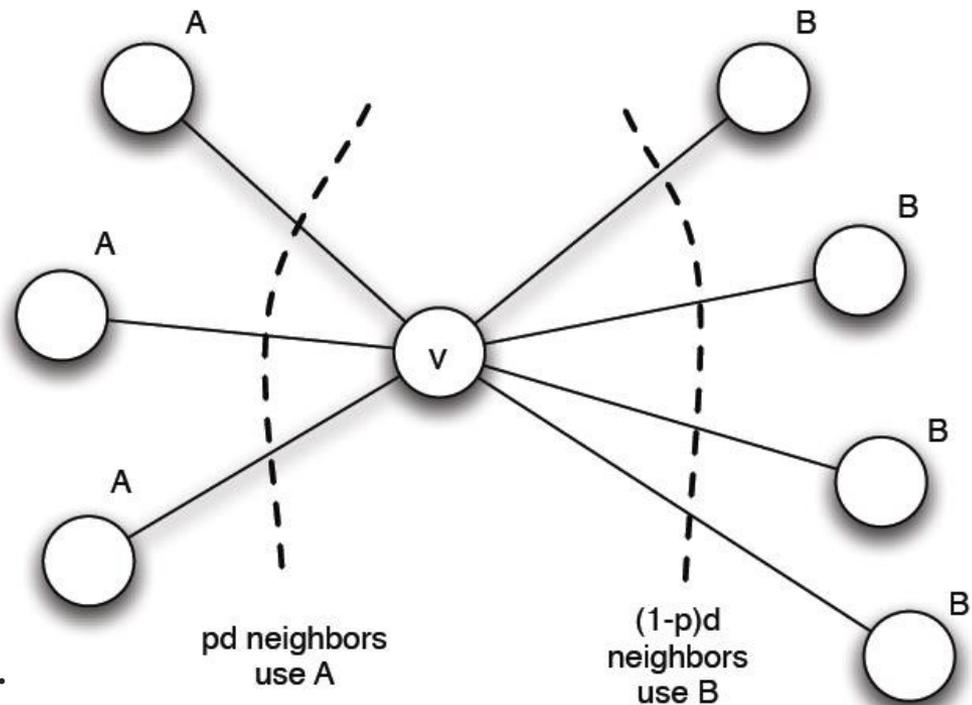
# Diffusion- Cnt.

- $p$  fraction of  $v$ 's neighbors choose A
- $(1 - p)$  fraction choose B.
- $v$  has  $d$  neighbors
- Which behavior should  $v$  adopt?



# Diffusion- Cnt.

- $p$  fraction of  $v$ 's neighbors choose A
- $(1 - p)$  fraction choose B.
- $v$  has  $d$  neighbors
  - If  $v$  chooses A
    - payoff =  $p \times d \times a$
  - If  $v$  chooses B
    - payoff =  $(1 - p) \times d \times b$



$$pda \geq (1 - p)db, \quad p \geq \frac{b}{a + b}.$$

$$q = \frac{b}{a + b}$$

$$q = 0.05 \quad \text{vs} \quad q = 0.95$$

# Diffusion- Cnt.

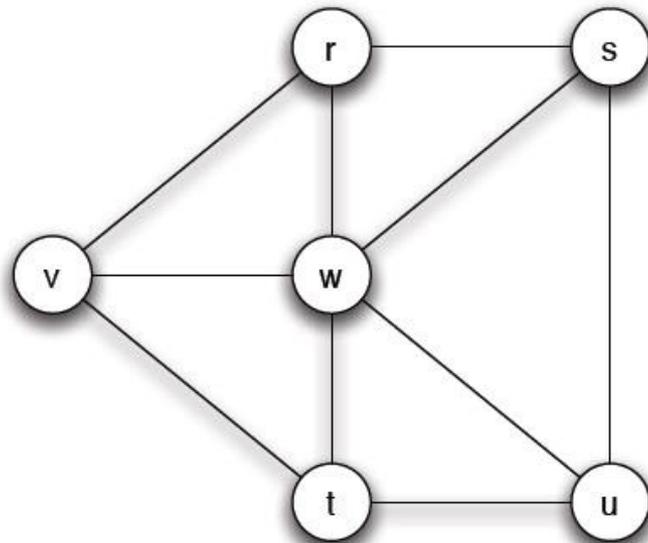
- Cascading behavior
  - Everyone adopts A,
  - Everyone adopts B,
  - **Intermediate state: some adopt A and some adopt B!**

# Diffusion- Cnt.

- Suppose everyone initially use B as a default behavior.
- A small set of **initial adopters** decide to switch to A.
- Cascade may start:
  - some neighbors of initial adopters may switch to A, then their neighbors, and so forth
- Cascade stops if:
  - **Complete cascade**: every node switch over to A!
  - We reach a step where no node wants to switch!  
(coexistence btw A and B)
- That depends on:
  - the network structure,
  - the choice of initial adopters,
  - the value of the threshold  $q$

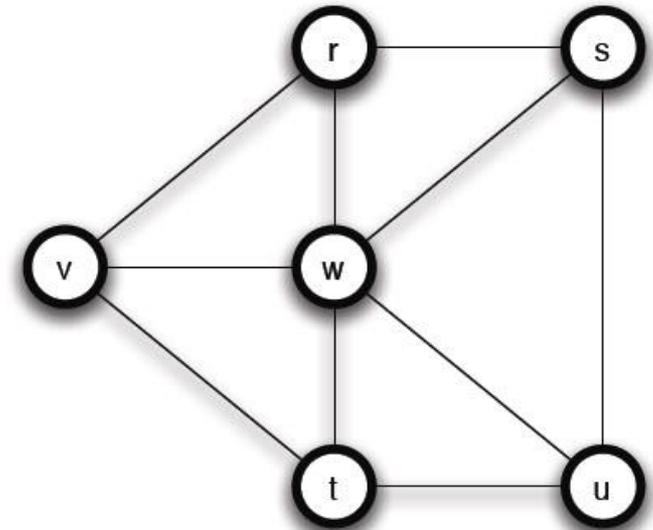
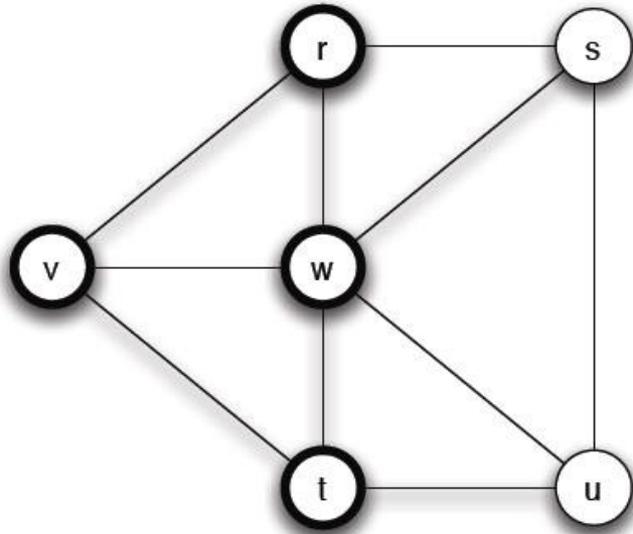
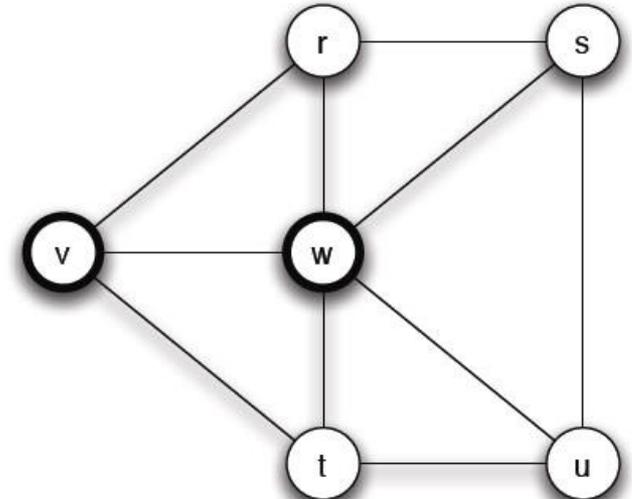
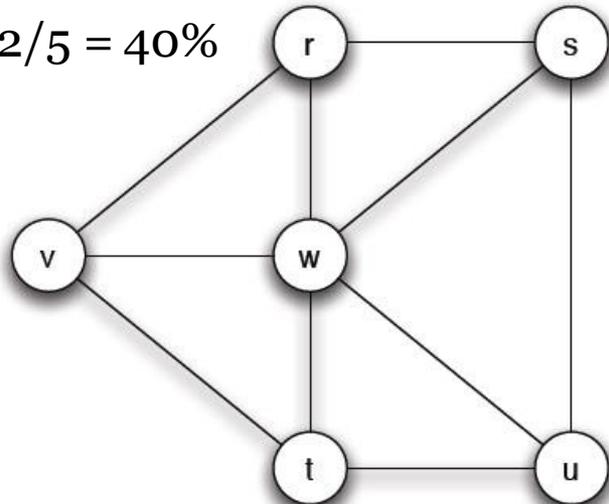
# Diffusion- Cnt.

- Payoff  $a=3$  and  $b=2$ .
- $q = 2/5$ , nodes switch to A if at least 40% of their neighbors are using A!
- $v$  and  $w$  are initial adopters of A!



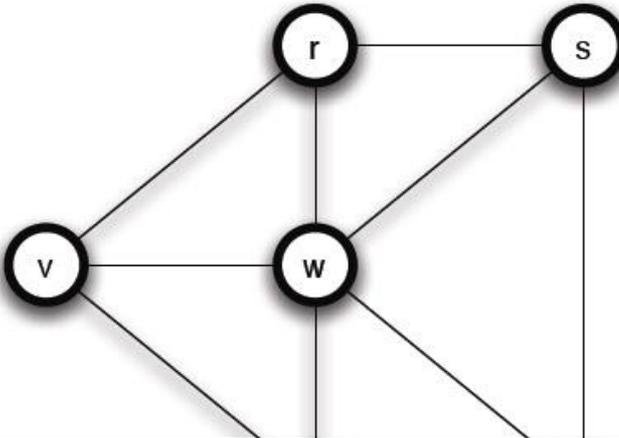
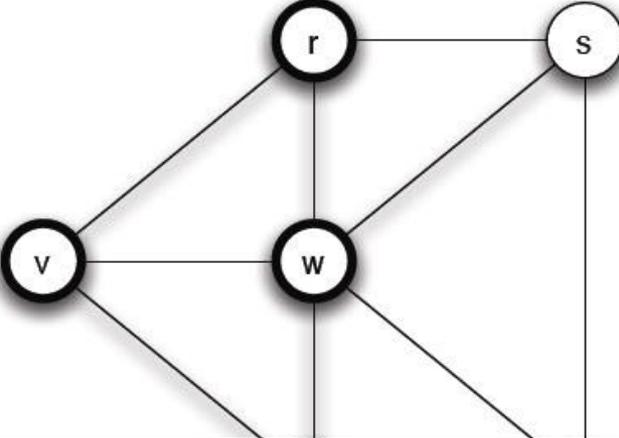
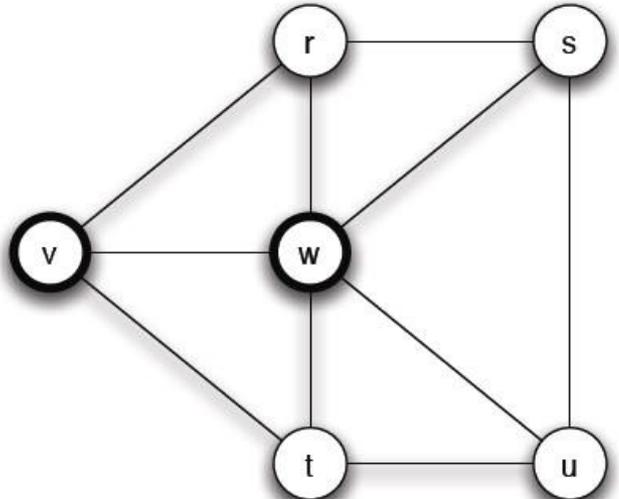
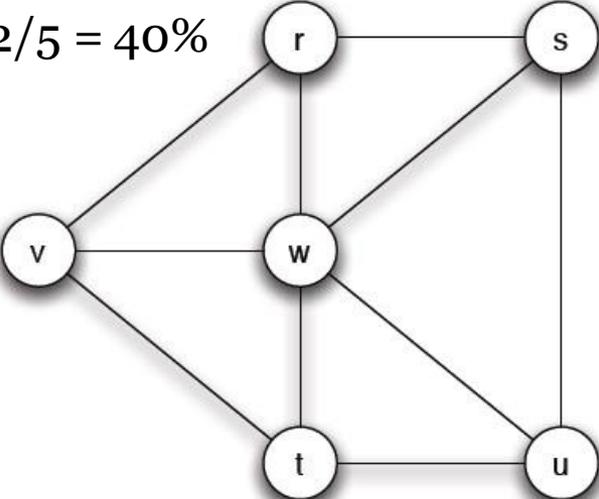
# Diffusion- Cnt.

$$q = 2/5 = 40\%$$



# Diffusion- Cnt.

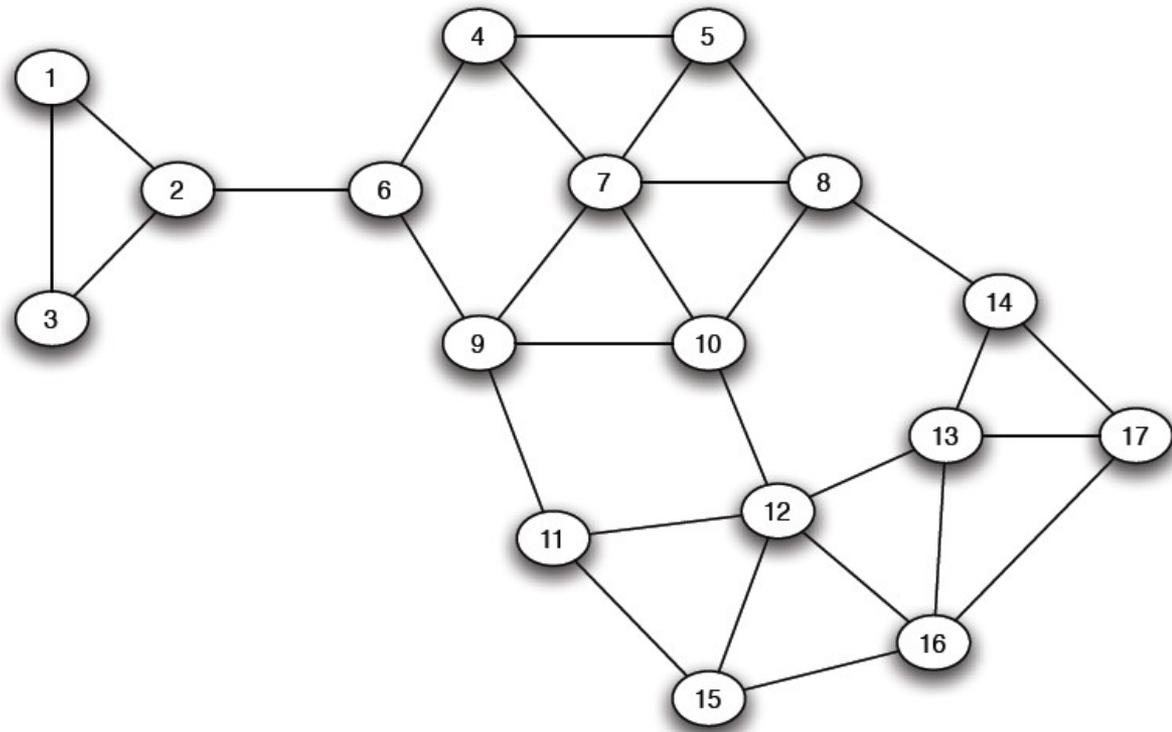
$q = 2/5 = 40\%$



**chain reaction:** *v* and *w* aren't able to get *s* and *u* to switch by themselves, but once they've converted *r* and *t*, this provides enough leverage.

# Diffusion- Cnt.

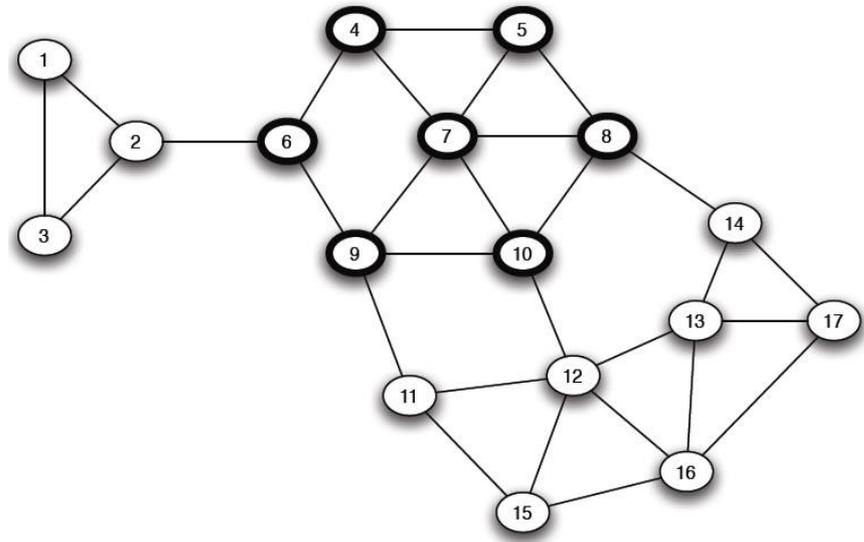
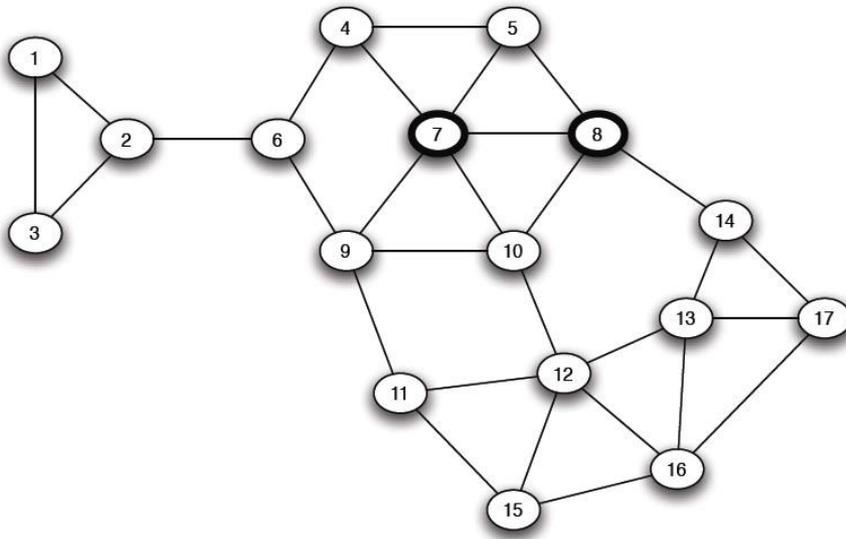
- $a=3$  and  $b=2$ .
- $q = 2/5$
- 7 and 8 are initial adopters of A!



# Diffusion- Cnt.

- Takes 3 steps for the cascade to stop!
  - 5 and 10 switch to A, then
  - nodes 4 and 9, then
  - node 6.
- No further nodes will be willing to switch!

$$q = 2/5$$



Tightly-knit communities in the network can hinder the spread of a behavior.

# Diffusion- Cnt.

- What are useful strategies to push adoption of A (assume A and B are competing technologies)?

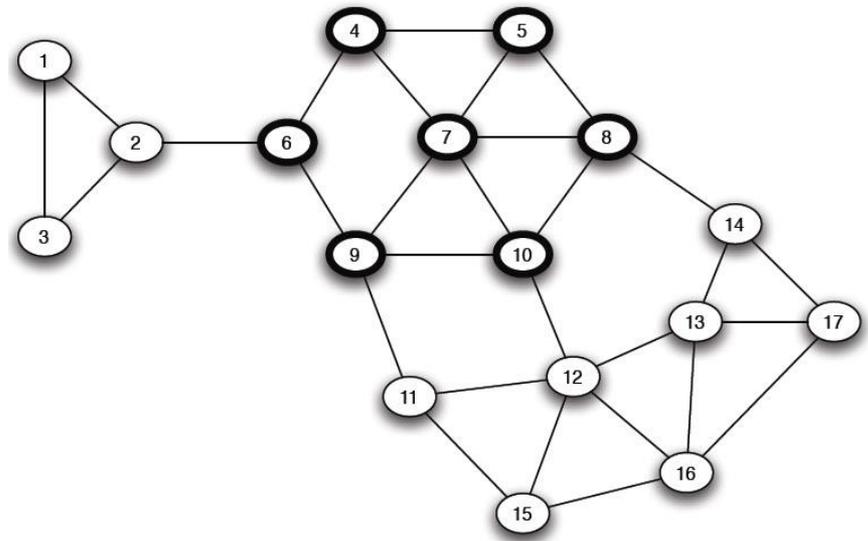
# Diffusion- Cnt.

- Strategies that are useful to push adoption of A
  - Change the payoff
    - Say from  $a = 3$  to  $a = 4$ !
    - $q$  drops from  $2/5$  down to  $1/3$ 
      - then all nodes will switch to A in the above example.

$$q = b/(a+b).$$

# Diffusion- Cnt.

- Strategies that are useful to push adoption of A
  - Convince a small number of key nodes in the part of the network using B to switch to A
    - Choose carefully so as to get the cascade going again!
    - Convince 12?
    - Convince 14?



# Lecture Topics

- Modeling Diffusion
- **Cascades & Clusters**
- Cascade Capacity
- Collective Action

# Cascades & Clusters

- Question: What makes a cascade stop? Or prevents it from breaking into all parts of a network?

# Cascades & Clusters

- Question: What makes a cascade stop? Or prevents it from breaking into all parts of a network?
  - A cascade comes to stop when it runs into a **dense cluster** (tightly-knit **communities** & **homophily**),
  - This is the **only** thing that causes cascades to stop!

# Cascades & Clusters- Cnt.

- **Cluster Density**

- A cluster with density  $p$  is a set of nodes such that each node has **at least**  $p$  fraction of its neighbors in the set.

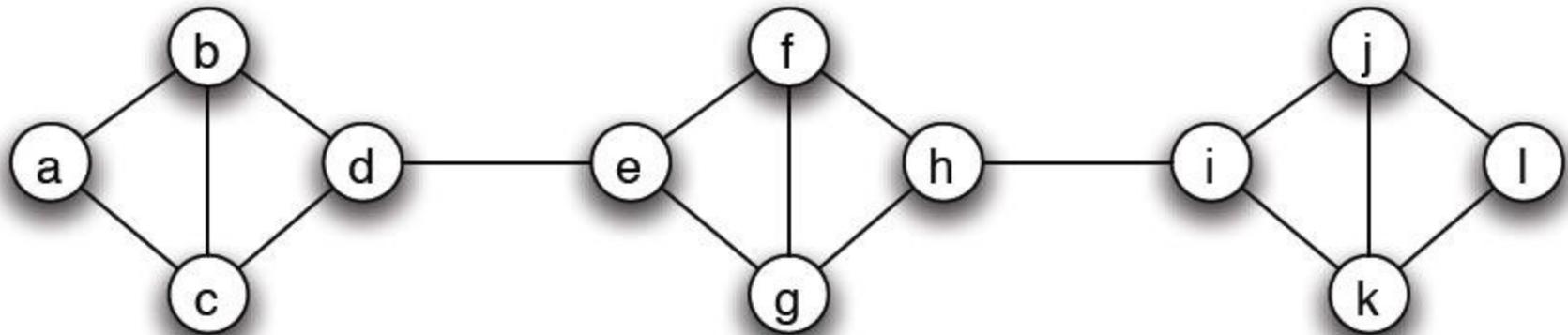
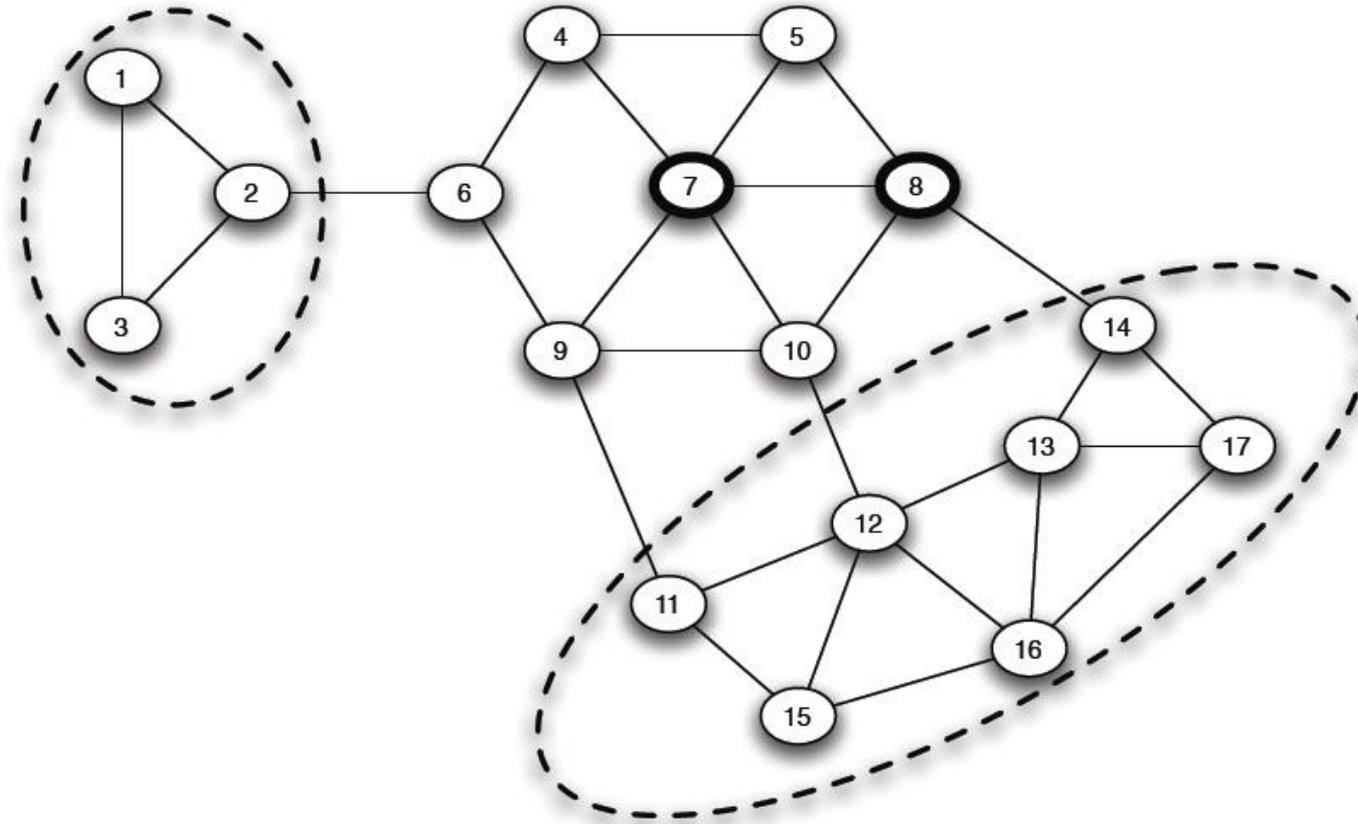


Figure 19.6: A collection of four-node clusters, each of density  $2/3$ .

# Cascades & Clusters- Cnt.

- **Claim:** Given initial adopters of A & threshold  $q$ :
  - i. If remaining network contains a cluster of density greater than  $1 - q$ , then no complete cascade.
  - ii. If there is no complete cascade, the remaining network contains a cluster of density  $> 1 - q$ .

# Cascades & Clusters- Cnt.



$$q = 2/5 = 40\%$$

$$\text{Cluster density} = 2/3 = 66\%$$

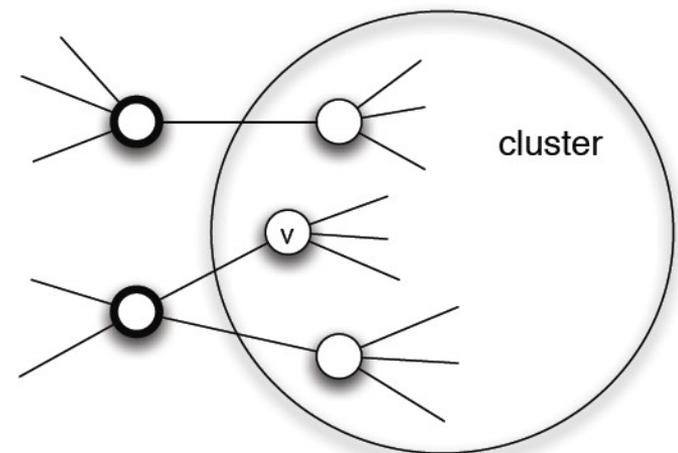
$$\text{Cluster density} > (1-q) = 60\%$$

# Cascades & Clusters- Cnt.

- i. If remaining network contains a cluster of density greater than  $1 - q$ , then no complete cascade.

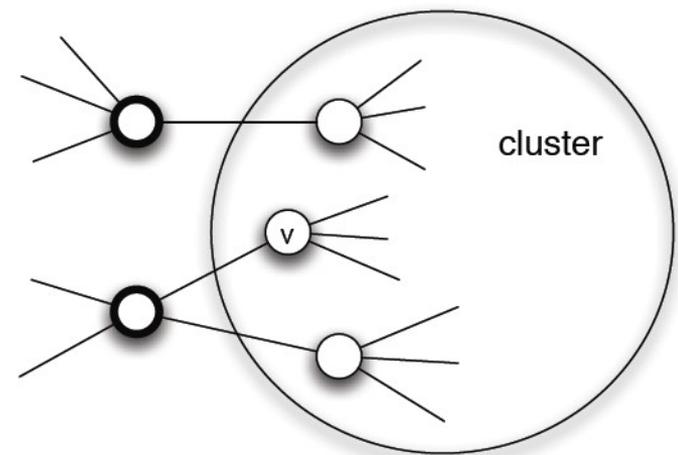
# Cascades & Clusters- Cnt.

- i. If remaining network contains a cluster of density greater than  $1 - q$ , then no complete cascade.
- **Solution**
  - Assume there is a node inside the cluster (density  $> 1 - q$ ) that adopts A
  - Let  $v$  be the **first** node that does so.



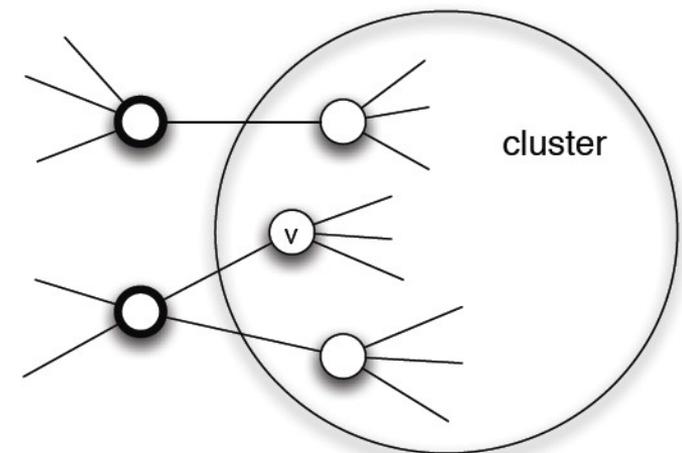
# Cascades & Clusters- Cnt.

- i. If remaining network contains a cluster of density greater than  $1 - q$ , then no complete cascade.
- **Solution**
  - The only neighbors of  $v$  that were using A at the time it decided to switch were **outside** the cluster.



# Cascades & Clusters- Cnt.

- i. If remaining network contains a cluster of density greater than  $1 - q$ , then no complete cascade.
- **Solution**
  - But, more than a  $1-q$  fraction of  $v$ 's neighbors are inside the cluster,
  - Thus less than a  $q$  fraction of  $v$ 's neighbors are outside the cluster.
  - Thus  $v$  cannot adopt A



clusters block the spread of cascades

# Cascades & Clusters- Cnt.

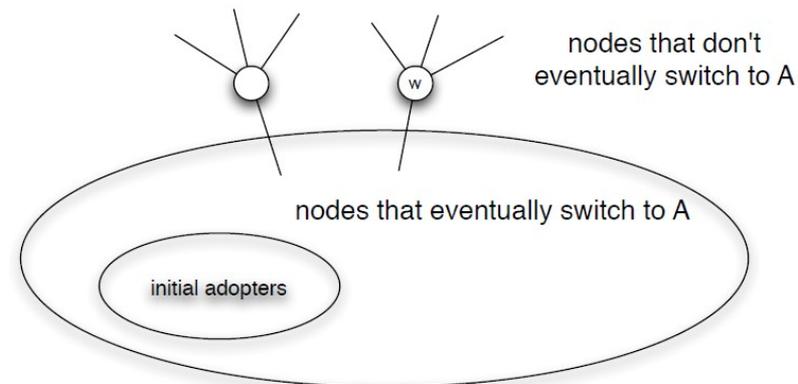
- ii. If there is no complete cascade, the remaining network contains a cluster of density  $> 1 - q$ .

# Cascades & Clusters- Cnt.

ii. If there is no complete cascade, the remaining network contains a cluster of density  $> 1 - q$ .

- **Solution**

- Run the process until it stops!
  - there are nodes using B that don't want to switch.
  - let  $S$  denote such nodes.



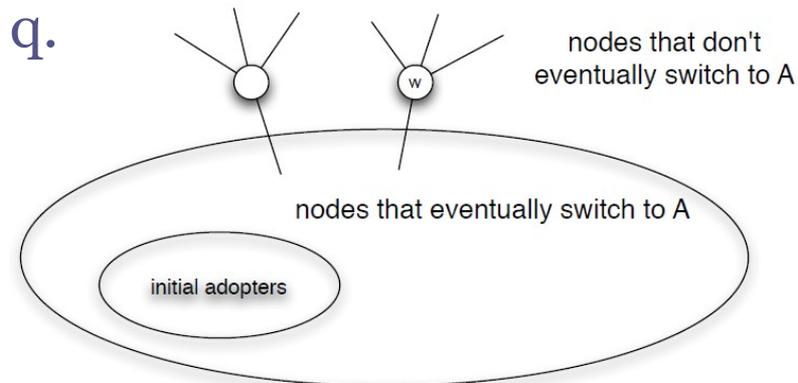
# Cascades & Clusters- Cnt.

ii. If there is no complete cascade, the remaining network contains a cluster of density  $> 1 - q$ .

- **Solution**

- Run the process until it stops!

- consider any node  $w \in S$
    - fraction of  $w$ 's neighbors using A is  $< q$ .
    - fraction of  $w$ 's neighbors using B is  $> 1 - q$ .
    - This holds for any node  $w \in S$ 
      - $S$  is a cluster of density  $> 1 - q$ .



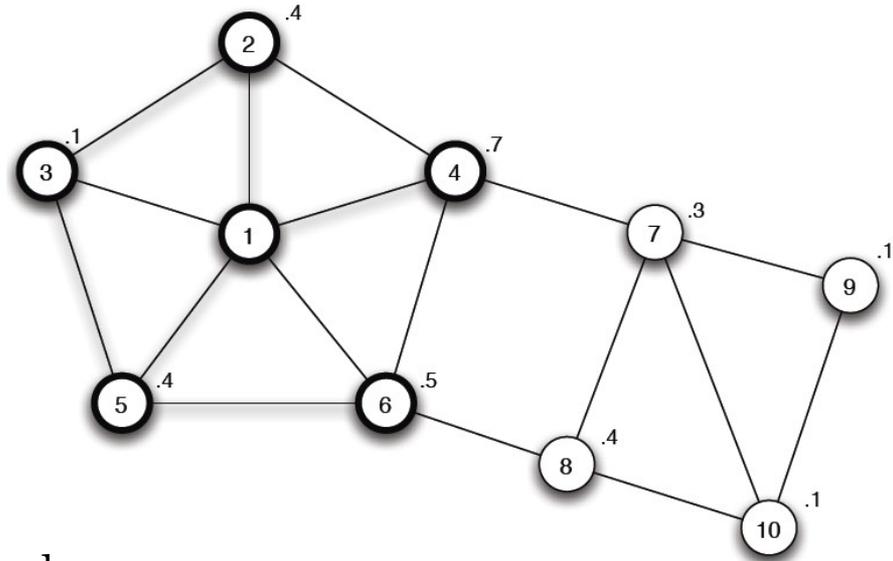
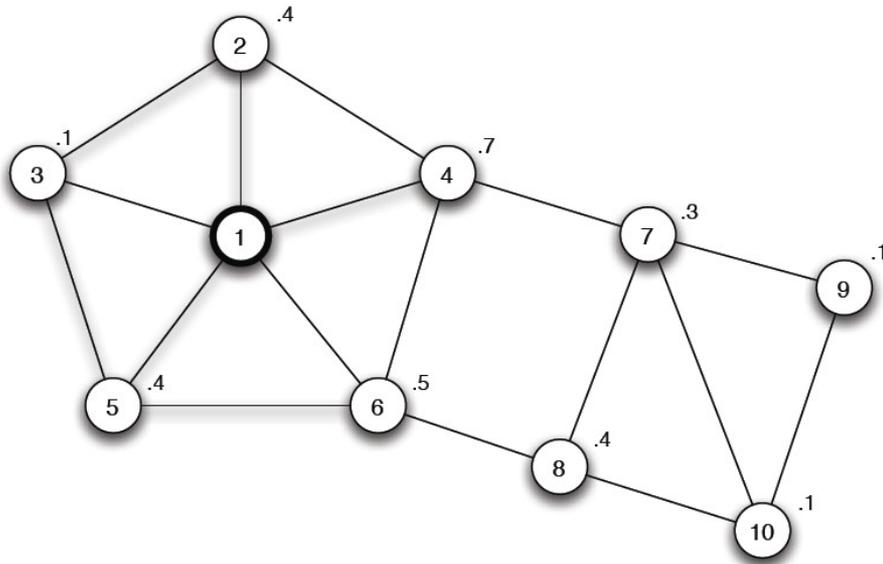
Whenever a cascade comes to a stop, there's a cluster that can be used to explain why.

# Extensions of Cascade Model

- **Heterogeneous thresholds**
  - each nodes  $v$  has a **node-specific** threshold ( $q_v$ ) for adopting a behavior!
- $v$  has  $d$  neighbors of whom a  $p$  fraction have behavior A, and a  $(1 - p)$  fraction have behavior B:
  - Payoff from choosing A is  $pda_v$
  - Payoff from choosing B is  $(1 - p)db_v$ .
- A is better for  $v$  if
  - $pda_v > (1 - p)db_v$ .

$$p \geq \frac{b_v}{a_v + b_v}.$$

# Extensions of Cascade Model- Cnt.



- Without node-specific thresholds, there would no cascade.
- The extremely low threshold of node 3 lead to diffusion.

The power of **influential nodes** is correlated to the extent to which such nodes have access to easily **influenceable nodes**.

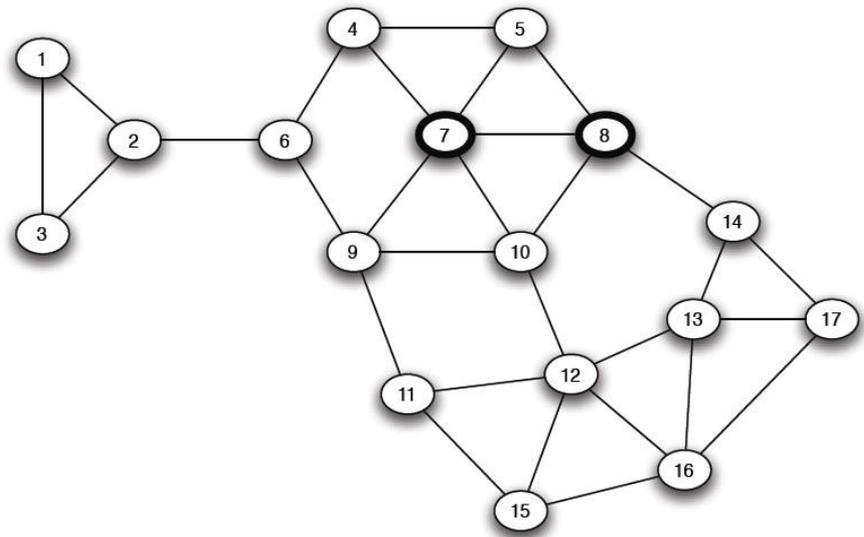
- Clusters are still obstacle to cascades
- A **blocking cluster** is a set of nodes for which each node  $v$  has  $> 1 - q_v$  fraction of its neighbors in the set.
  - Heterogeneous cluster density: node-specific threshold for the fraction of friends to have in cluster.

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- Modeling Diffusion
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# Cascade Capacity

- **Cascade capacity** of a network: The maximum  $q$  for which some **small** set (*finite set*) of initial adopters can cause a **complete cascade**!
  - Indicates how different network structures are **hospitable** to cascades!



# Cascade Capacity- Cnt.

- Let  $S$  be the small set of early adopters of  $A$ .
- What is cascade capacity?
  - the maximum  $q$  for complete cascade?



# Cascade Capacity- Cnt.

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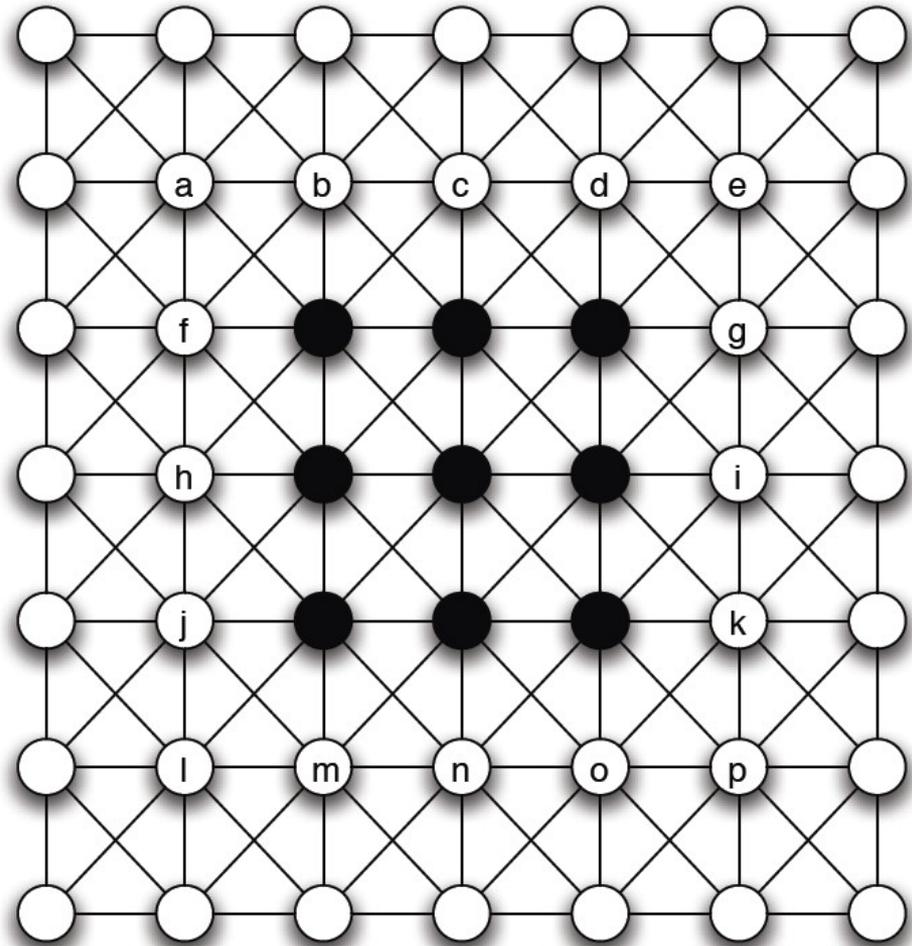


If  $q \leq 1/2$ , complete cascade.

If  $q > 1/2$ , no finite set of initial adopters can get any node to switch to  $A$ .

**Cascade capacity =  $1/2$**

# Cascade Capacity- Cnt.

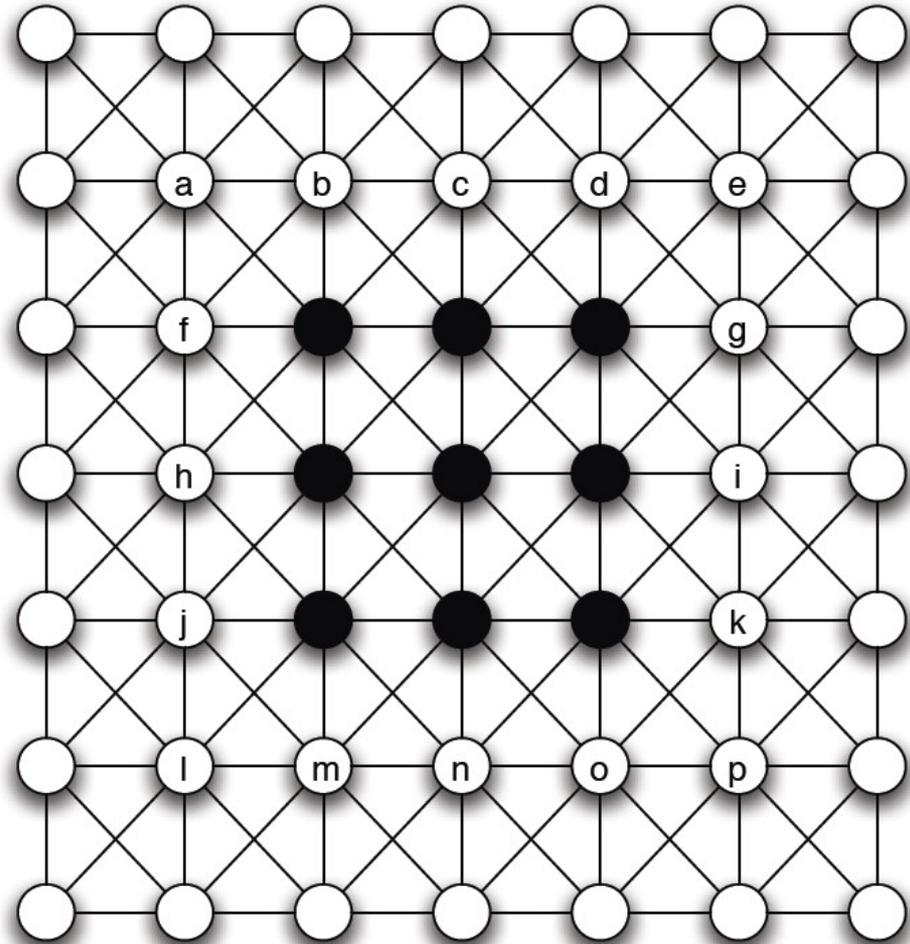


# Cascade Capacity- Cnt.

If  $q \leq 3/8$ , then there is a complete cascade: first to the nodes  $c, h, i, n$ ; then to nodes  $b, d, f, g, j, k, m, o$ ; and then to others

If  $q > 3/8$ , no node will choose to adopt A.

**Cascade Capacity=3/8**



# Cascade Capacity- Cnt.

- How easy cascades propagate in a network with *large* cascade capacity?

# Cascade Capacity- Cnt.

- How easy cascades propagate in a network with *large* cascade capacity?
  - Cascades happen more “easily”!
  - they happen even for behaviors A that don't offer much payoff advantage over the default behavior B.

# Cascade Capacity- Cnt.

- What is the maximum possible value of cascade capacity?

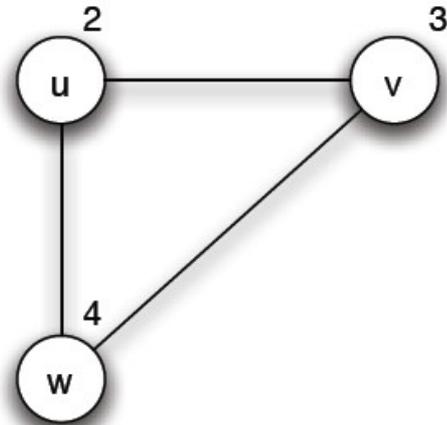
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# Collective Action

- Collective action
  - Consider a protest against something that we are good at complaining about 😊
  - Each node  $v$  shows up if at least  $v_k$  people (including itself) show up.
  - Each person knows the thresholds of her neighbors and structure of the net.
- Given a network with a set of thresholds, is the protest likely to happen?

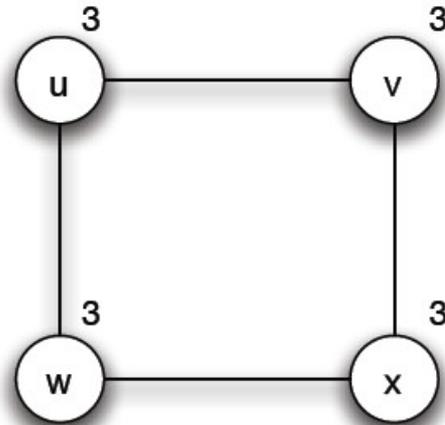
# Collective Action- Cnt.



(a) *An uprising will not occur*

- $w$  never join
- $v$  knows  $w$ 's threshold, so  $v$  knows that  $w$  won't participate.
- $v$ 's threshold is 3,  $v$  won't participate either.

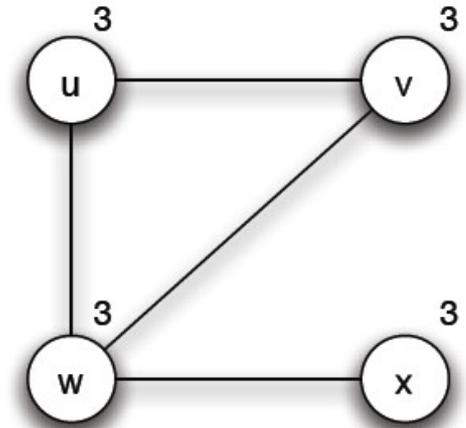
# Collective Action- Cnt.



(b) *An uprising will not occur*

- $u$  doesn't know  $x$ 's threshold
- $x$ 's threshold could be high, like 5.  $\rightarrow v$  and  $w$  don't join  $\rightarrow$  if  $u$  join, she'd be the only one.
- Each node knows there are 3 nodes with thresholds 3 (enough to form protest) but they hold back as they are not sure that other nodes know this.

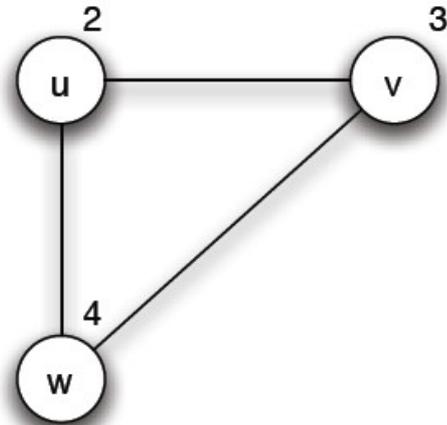
# Collective Action- Cnt.



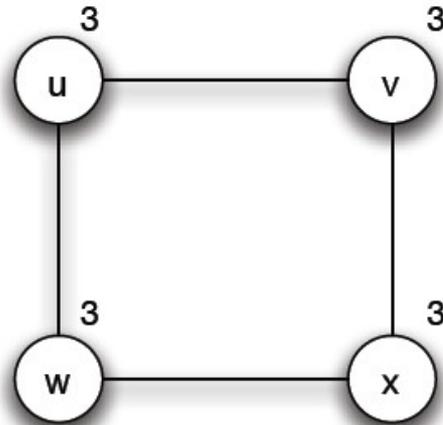
(c) *An uprising can occur*

- each of  $u$ ,  $v$ , and  $w$  knows that there are 3 nodes with thresholds of 3
- And this fact is “common knowledge”

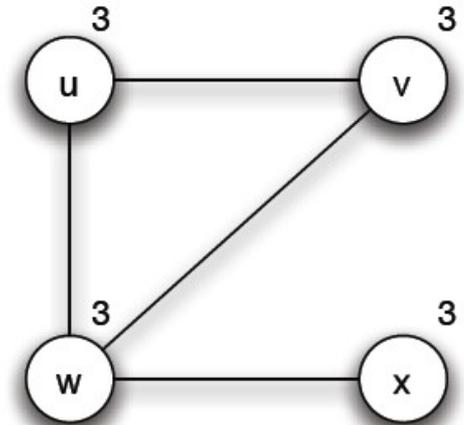
# Collective Action- Cnt.



(a) *An uprising will not occur*



(b) *An uprising will not occur*



(c) *An uprising can occur*

Social nets allow individuals to base decisions on what others know.

# Information diffusion on Twitter

- <https://snikolov.wordpress.com/2012/11/12/information-diffusion-on-twitter/>

# Reading

- Ch.19 Cascading Behavior in Networks [NCM]